CORRELATED $\pi\pi$ AND $K\overline{K}$ EXCHANGE IN THE BARYON-BARYON INTERACTION

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Abstract

The exchange of two correlated pions or kaons provides the main part of the intermediate-range attraction between two baryons. Here, a dynamical model for correlated two-pion and two-kaon exchange in the baryon-baryon interaction is presented, both in the scalar-isoscalar (σ) and the vector-isovector (ρ) channel. The contribution of correlated $\pi\pi$ and $K\overline{K}$ exchange is derived from the amplitudes for the transition of a baryon-antibaryon state ($B\overline{B'}$) to a $\pi\pi$ or $K\overline{K}$ state in the pseudophysical region by applying dispersion theory and unitarity. For the $B\overline{B'} \to \pi\pi, K\overline{K}$ amplitudes a microscopic model is constructed, which is based on the hadron-exchange picture. The Born terms include contributions from baryon-exchange as well as ρ-pole diagrams. The correlations between the two pseudoscalar mesons are taken into account by means of $\pi\pi$-$K\overline{K}$ amplitudes derived likewise from a meson-exchange model, which is in line with the empirical $\pi\pi$ data. The parameters of the $B\overline{B'} \to \pi\pi, K\overline{K}$ model, which are related to each other by the assumption of $SU(3)$ symmetry, are determined by the adjustment to the quasiempirical $N\overline{N} \to \pi\pi$ amplitudes in the pseudophysical region. It is found that correlated $K\overline{K}$ exchange plays an important role in the σ-channel for baryon-baryon states with non-vanishing strangeness. The strength of correlated $\pi\pi$ plus $K\overline{K}$ exchange in the σ-channel decreases with the strangeness of the baryon-baryon system becoming more negative. Due to the admixture of baryon-exchange processes to the $SU(3)$-symmetric ρ-pole contributions the results for correlated $\pi\pi$-exchange in the vector-isovector channel deviate from what is expected in the naive $SU(3)$ picture for genuine ρ-exchange. In present models of the hyperon-nucleon interaction contributions of correlated $\pi\pi$ and $K\overline{K}$ exchange are parametrized for simplicity by single σ and ρ exchange. Shortcomings of this effective description, e.g. the missing long-range contributions, are pointed out by comparison with the dispersiontheoretic results.

1 Introduction

The study of the role of strangeness degrees of freedom in low energy nuclear physics is of high current interest since it should lead to a deeper understanding of the relevant

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strong interaction mechanisms in the non-perturbative regime of QCD. For example, the system of a strange baryon (hyperon Y) and a nucleon (N) is in principle an ideal testing ground to investigate the importance of $SU(3)_{\rm flavor}$ symmetry for the hadronic interactions. This symmetry is obviously broken already by the different masses of hadrons sitting in the same multiplet. However the important question arises whether (on the level of hadrons) it is broken not only kinematically but also dynamically, e.g. in the values of the coupling constants at the hadronic vertices. The answer cannot be given at the moment since the present empirical information about the YN interaction is too scarce and thus prevents any definite conclusions. Hopefully the situation will be improved by experiments of elastic $\Sigma^{\pm}p$, Λp, and even Ξp, currently performed at KEK [1, 2].

Existing meson exchange models of the YN interaction assume for the hadronic coupling constants at least $SU(3)$ symmetry, in case of models A and B of the Jülich group [3] even $SU(6)$ of the static quark model. This symmetry requirement provides relations between coupling constants of a meson multiplet to the baryon current, which strongly reduce the number of free model parameters. Specifically, coupling constants at the strange vertices are then connected to nucleon-nucleon-meson coupling constants, which in turn are fixed between close boundaries by the wealth of empirical NN scattering information. All YN interaction models can reproduce the existing empirical YN scattering data. Therefore at present the assumption of SU(3) symmetry for the coupling constants is not in conflict with experiment.

However, the treatment of the scalar-isoscalar meson sector, which provides the intermediate range baryon-baryon interaction, is conceptionally not very convincing so far. The one-boson-exchange models of the Nijmegen group start from the existence of a broad scalar-isoscalar $\pi\pi$ resonance (ϵ-meson, $m_\epsilon = 760\,MeV$, $\Gamma_\epsilon = 640\,MeV$), which is hidden in the experiment (e.g. $\pi N \to \pi\pi N$) under the strong ρ^0-signal and can therefore not be identified reliably. For practical reasons the exchange of this broad ϵ-meson is then approximated by the exchange of a sum of two mesons with sharp mass m_1 and m_2; the smaller mass is around $500\,MeV$ and thus corresponds to the phenomenological σ-meson in conventional OBE-models. The ϵ-meson is then treated as $SU(3)$ singlet (model D [4]) or as member of a full nonet of scalar mesons (model F [5] and NSC [6]). In the $SU(3)$ framework the ϵ-coupling strength is equal for all baryons in model D, while it depends on four open $SU(3)$ parameters in models F and NSC, which are adjusted to NN and YN scattering data. In the latest Nijmegen model NSC [6] the scalar meson nonet includes apart from the ϵ the isoscalar $f_0(975)$, the isovector $a_0(980)$ and the strange mesons κ, which the authors identify [7] with scalar $q^2\bar{q}^2$-states predicted by the MIT bag model [8]. This interpretation is however doubtful, at least for the $f_0(975)$ and $a_0(980)$. According to a recent theoretical analysis of the $\pi\pi$, $\pi\eta$, and $K\overline{K}$ system [9] in the meson exchange framework the $f_0(975)$ is a $K\overline{K}$ molecule bound in the $\pi\pi$ continuum, while $a_0(980)$ is dynamically generated by the $K\overline{K}$ threshold. Thus both mesons do not appear to be genuine quark model resonances, with the consequence that $SU(3)$ relations should not be applied to these mesons. (The non-strange members of the scalar nonet are expected to be at higher energies).

In the Bonn potential [10] the intermediate range attraction is provided by uncorrelated (Fig. 1a,b) and correlated (Fig. 1c) $\pi\pi$ exchange processes with NN, $N\Delta$ and $\Delta\Delta$ intermediate states. It is known from the study of the $\pi\pi$ interaction that the $\pi\pi$ correlations are important mainly in the scalar-isoscalar and vector-isovector channel.

The Bonn potential includes such correlations, however only in a rough way, namely in terms of sharp mass σ' and ρ exchange. One disadvantage of such a simplified treatment is that this parametrization cannot be transported into the hyperon sector in a well defined way. Therefore in the YN interaction models of the Jülich group [3], which start from the Bonn NN potential, the coupling constants of the fictitious σ'-meson at the strange vertices ($\Lambda\Lambda\sigma'$, $\Sigma\Sigma\sigma'$) are essentially free parameters. In view of the little empirical information about the YN interaction this feature is not satisfactory. This is especially true for an extension of the YN models to baryon-baryon channels with strangeness $S = -2$. So far there is no empirical information about these channels (apart from some data on Ξ- and $\Lambda\Lambda$-hypernuclei). Still there is a large interest in these channels initiated by the prediction of the H-dibaryon by Jaffe [11]. The H-dibaryon is a deeply bound 6-quark state with the same quark content as the $\Lambda\Lambda$ system (*uuddss*) and with $^1S_0$ quantum numbers. For the experimental search it is important to know whether conventional deuteron-like $\Lambda\Lambda$ states exist. An analysis of possible $S = -2$ bound states in the meson exchange framework could provide valuable information in this regard, but requires a coupled channels treatment of $\Lambda\Lambda$, $\Sigma\Sigma$, and $N\Xi$ channels. An extension of the Jülich YN models to those channels is only of minor predictive power since the strength of the important $\Xi\Xi\sigma'$ vertex is completely undetermined and cannot be fixed by empirical data.

These problems can be overcome by an explicit evaluation of correlated $\pi\pi$ exchange processes in the various baryon-baryon channels. A corresponding calculation has been done for the NN case (Fig. 1c) [12]. Starting point was a fieldtheoretic model for both the $N\overline{N} \to \pi\pi$ Born amplitudes and the $\pi\pi$-$K\overline{K}$ interaction [13]. With the help of unitarity and dispersion relations the amplitude for the correlated $\pi\pi$ exchange in the NN interaction has been determined, showing characteristic discrepancies to σ' and ρ exchange in the (full) Bonn potential.

For the correct description of the $\pi\pi$ interaction in the scalar-isoscalar channel the coupling to the $K\overline{K}$ channel is essential, which is obvious from the interpretation of the $f_0(975)$ as a $K\overline{K}$ bound state. Apart from the $\pi\pi$-$K\overline{K}$ interaction model the $K\overline{K}$ channel is not considered in Ref. [12], i.e. the coupling of the kaon to the nucleon is not taken into account. In fact, this approximation is justified in the NN system [14]; it is however not expected to work in channels involving hyperons.

The aim of the present paper is a microscopic derivation of correlated $\pi\pi$ as well as $K\overline{K}$ exchange processes in the various baryon-baryon channels with $S = 0, -1, -2$ (Fig. 2). The $K\overline{K}$ channel is treated on an equal footing with the $\pi\pi$ channel in order to determine reliably the influence of $K\overline{K}$ correlations. Our results replace the phenomenological σ' and ρ exchange in the Bonn NN and Jülich YN models by correlated processes and in this way eliminate undetermined model parameters (e.g. σ' coupling constants). Corresponding interaction models thus have more predictive power and should make a sensible treatment of $S = -2$ baryon-baryon channels possible.

The formal treatment is similar to that of Refs. [12, 15, 16] dealing with correlated $\pi\pi$ exchange in the NN interaction. Due to the inclusion of the $K\overline{K}$ channel and different baryon masses (e.g. in the $N\Lambda$ channel) generalizations are however required at some places. Starting point is a field-theoretic model for the baryon-antibaryon ($B\overline{B'}$) $\to \pi\pi$, $K\overline{K}$ Born amplitudes in the $J^P = 0^+, 1^-$ channels. Besides various baryon exchange terms the model includes in complete consistency to the $\pi\pi$-$K\overline{K}$ interaction model [13, 17] also a ρ-pole term (cf. Fig. 3). These Born amplitudes are analytically

continued into the pseudophysical region below the $B\overline{B'}$-threshold. The solution of a covariant scattering equation with full inclusion of $\pi\pi - K\overline{K}$ correlations yields the $B\overline{B'} \to \pi\pi$, $K\overline{K}$ amplitudes in the pseudophysical region. In the $N\overline{N} \to \pi\pi$ channel these amplitudes are then adjusted to quasiempirical information [18, 19], which has been obtained by analytic continuation of πN and $\pi\pi$ data. With the assumption of $SU(6)$ symmetry for the coupling constants a parameter-free description of the other particle channels can then be achieved.

Via unitarity relations the products of $B\overline{B'} \to \pi\pi$, $K\overline{K}$ amplitudes fix the singularity structure of the baryon-baryon amplitudes for $\pi\pi$ and $K\overline{K}$ exchange. Assuming analyticity for the amplitudes dispersion relations can be formulated for the baryon-baryon amplitudes, which connect physical amplitudes in the s-channel with singularities and discontinuities of these amplitudes in the pseudophysical region of the t-channel processes. With a suitable subtraction of uncorrelated contributions, which are calculated directly in the s-channel and therefore guaranteed to have the correct energy behavior, we finally obtain the amplitudes for correlated $\pi\pi$ and $K\overline{K}$ exchange in the baryon-baryon system.

In the next chapter we describe the underlying formalism which is used to derive correlated $\pi\pi$ and $K\overline{K}$ exchange potentials for the baryon–baryon amplitudes. Furthermore we present our microscopic model for the required $B\overline{B'} \to \pi\pi$, $K\overline{K}$ amplitudes. Sect.3 contains our results and also a comparison with those obtained from other models. The paper ends with some concluding remarks.

2 Formalism

2.1 Kinematics and amplitudes

The kinematics of a two-body scattering process $A+B \to C+D$ (cf. Fig. 4) is uniquely determined by the 4-momenta p_A, p_B, p_C, p_D of the particles. Taking into account the on-mass-shell relations ($p_X^2 = M_X^2, X = A, \ldots, D$) and the conservation of the total 4-momentum ($p_A + p_B = p_C + p_D$) only two independent Lorentz-scalars can be built out of these momenta. For these Lorentz-scalars one usually introduces the three Mandelstam variables

$$\begin{aligned} s &= (p_A + p_B)^2 &= (p_C + p_D)^2 \quad , \\ t &= (p_C - p_A)^2 &= (p_B - p_D)^2 \quad , \\ u &= (p_D - p_A)^2 &= (p_B - p_C)^2 \quad , \end{aligned} \tag{1}$$

which are related by

$$s + t + u = M_A^2 + M_B^2 + M_C^2 + M_D^2 \equiv \Sigma \quad . \tag{2}$$

By crossing the scattering process $A + B \to C + D$ is closely related to two other processes, as indicated by Fig. 4:

$$\begin{array}{lclclcll} A, p_a & + & B, p_B & \to & C, p_C & + & D, p_D & \text{‘}s\text{-channel’}\,, \\ A, p_A & + & \overline{C}, -p_C & \to & \overline{B}, -p_B & + & D, p_D & \text{‘}t\text{-channel’}\,, \\ A, p_A & + & \overline{D}, -p_D & \to & C, p_C & + & \overline{B}, -p_B & \text{‘}u\text{-channel’}\,. \end{array} \tag{3}$$

Here, the channels are named according to the Mandelstam variable which denotes the squared total energy in the center-of-mass (c.m.) system.

For the s-channel process the particle 4-momenta in the c.m. system read:

$$p_A = \begin{pmatrix} E_A \\ \vec{p}_s \end{pmatrix}, \quad p_B = \begin{pmatrix} E_B \\ -\vec{p}_s \end{pmatrix}, \qquad p_C = \begin{pmatrix} E_C \\ \vec{q}_s \end{pmatrix}, \quad p_D = \begin{pmatrix} E_D \\ -\vec{q}_s \end{pmatrix} \tag{4}$$

with $E_X = \sqrt{M_X^2 + \vec{p}_X^{\,2}}$. The modulus of the relative momentum $\vec{p}_s$ ($\vec{q}_s$) of the initial (final) state can be expressed in terms of s:

$$\begin{aligned} \vec{p}_s^{\;2} &= \frac{\left[s - (M_A + M_B)^2\right]\,\left[s - (M_A - M_B)^2\right]}{4s} \quad , \\ \vec{q}_s^{\;2} &= \frac{\left[s - (M_C + M_D)^2\right]\,\left[s - (M_C - M_D)^2\right]}{4s} \quad . \end{aligned} \tag{5}$$

The scattering angle $\vartheta_s = \sphericalangle\,(\vec{p}_s, \vec{q}_s)$ is related to the Mandelstam variables by

$$\cos\vartheta_s = \frac{s(t-u) + (M_A^2 - M_B^2)(M_C^2 - M_D^2)}{4s|\vec{p}_s||\vec{q}_s|} \tag{6}$$

For the t-channel process $A + \overline{C} \to \overline{B} + D$ the c.m. 4-momenta of the particles are

$$p_A = \begin{pmatrix} E_A \\ \vec{p}_t \end{pmatrix}, \quad -p_C = \begin{pmatrix} E_C \\ -\vec{p}_t \end{pmatrix}, \qquad p_D = \begin{pmatrix} E_D \\ \vec{q}_t \end{pmatrix}, \quad -p_B = \begin{pmatrix} E_B \\ -\vec{q}_t \end{pmatrix}. \tag{7}$$

The analogue of Eqs. 5 for the modulus of the relative momenta $\vec{p}_t$, $\vec{q}_t$ and the scattering angle $\vartheta_t = \sphericalangle\,(\vec{p}_t, \vec{q}_t)$ now reads:

$$\begin{aligned} \vec{p}_t^{\;2} &= \frac{\left[t - (M_A + M_C)^2\right]\,\left[t - (M_A - M_C)^2\right]}{4t} \quad , \\ \vec{q}_t^{\;2} &= \frac{\left[t - (M_B + M_D)^2\right]\,\left[t - (M_B - M_D)^2\right]}{4t} \quad , \qquad (8) \\ \cos\vartheta_t &= \frac{t(u-s) + (M_A^2 - M_C^2)(M_D^2 - M_B^2)}{4t|\vec{p}_t||\vec{q}_t|} \quad . \qquad (9) \end{aligned}$$

Instead of the particle 4-momenta, usually the total 4-momentum and the following three linear combinations are used to characterize the kinematics of a two-body scattering process:

$$\Delta \equiv p_C - p_A = p_B - p_D, \quad P \equiv \frac{1}{2}(p_A + p_C), \quad Q \equiv \frac{1}{2}(p_B + p_D) \quad . \tag{10}$$

The scalar products of these three momenta can again be expressed in terms of the Mandelstam variables

$$\begin{aligned} \Delta^2 &= t \quad , & P\cdot\Delta &= \tfrac{1}{2}(M_C^2 - M_A^2) \quad , \\ P^2 &= \tfrac{1}{2}(M_A^2 + M_C^2) - t/4 \quad , & Q\cdot\Delta &= \tfrac{1}{2}(M_B^2 - M_D^2) \quad , \\ Q^2 &= \tfrac{1}{2}(M_B^2 + M_D^2) - t/4 \quad , & P\cdot Q &= (s-u)/4 \quad . \end{aligned} \tag{11}$$

In covariant field-theory the scattering amplitude T for a general process with n_i (n_f) particles in the initial (final) state $|\,i\,\rangle$ ($|\,f\,\rangle$) is related to the (unitary) S matrix by

$$\langle\, f\,|\,S\,|\,i\,\rangle = \langle\, f\,|\,i\,\rangle - i\frac{(2\pi)^4}{\sqrt{N_f N_i}}\delta^{(4)}(P_f - P_i)\langle\, f\,|\,T\,|\,i\,\rangle \quad , \tag{12}$$

where P_i (P_f) denotes the total 4-momentum in the initial (final) state. The factors N_x ($x = i, f$) are given by

$$N_x = (2\pi)^{3n_x} \prod_{j=1}^{n_x} \frac{2E_j}{(2M_j)^{b_j}} \quad , \tag{13}$$

where the j-th particle of channel x has mass M_j, momentum $\vec{p}_j$, energy $E_j = (M_j^2 + \vec{p}_j{}^2)^{1/2}$ and spin s_j. (Spin and isospin quantum numbers are suppressed for the moment.) The exponent b_j is given by

$$b_j = \begin{cases} 0 & \text{if } 2s_j \text{ even,} \\ 1 & \text{if } 2s_j \text{ odd.} \end{cases} \tag{14}$$

For a two-body scattering process $A + B \to C + D$ Eq. 12 reads

$$\begin{aligned} \langle\, Cp_C, Dp_D \,|\, S \,|\, Ap_A, Bp_B \,\rangle \;=\; & \langle\, Cp_C, Dp_D \,|\, Ap_A, Bp_B \,\rangle \\ - \frac{i}{(2\pi)^2} & \sqrt{\frac{(2M_A)^{b_A}(2M_B)^{b_B}(2M_C)^{b_C}(2M_D)^{b_D}}{16E_A E_B E_C E_D}}\,\delta^{(4)}(p_C + p_D - p_A - p_B) \\ & \langle\, Cp_C, Dp_D \,|\, T \,|\, Ap_A, Bp_B \,\rangle. \end{aligned} \tag{15}$$

Particles with spin (and helicity λ_X) are described in the helicity basis according to the conventions of Jacob and Wick [20]. By separating off the helicity spinors $u_X(\vec{p}_X, \lambda_X)$ of the particles from the scattering amplitudes one obtains the transition matrix $\mathcal{M}$. If, for instance, all four particles of the process $A+B \to C+D$ are spin-1/2 baryons, the transition matrix $\mathcal{M}$ is a 16×16 matrix in spinor space and is defined by

$$\begin{aligned} & \langle\, C\vec{p}_C\lambda_C, D\vec{p}_D\lambda_D \,|\, T \,|\, A\vec{p}_A\lambda_A, B\vec{p}_B\lambda_B \,\rangle = \\ & \quad \overline{u}_C(\vec{p}_C, \lambda_C)\overline{u}_D(\vec{p}_D, \lambda_D)\mathcal{M}_{AB\to CD}(P,Q)u_A(\vec{p}_A, \lambda_A)u_B(\vec{p}_B, \lambda_B) \quad . \end{aligned} \tag{16}$$

Now the transition matrix $\mathcal{M}$ can be constructed as a linear combination of the so-called *kinematic covariants* $\mathcal{O}_i$, which are like $\mathcal{M}$ operators in spinor space.

$$\mathcal{M}(P,Q) = \sum_i c_i(s,t)\mathcal{O}_i(P,Q) \quad . \tag{17}$$

The $\mathcal{O}_i$ are built up from the Dirac γ-matrices and the momenta P and Q (cf. Eq. 10) in such a way that their matrix elements are Lorentz-invariant quantities. The *invariant amplitudes* $c_i(s,t)$ are Lorentz-scalars.

The number of independent kinematic covariants for a given scattering process corresponds to the number of independent helicity amplitudes and is determined by the dimension of the spinor space and invariance principles for the underlying interaction. For the scattering of four spin-1/2 baryons ($A + B \to C + D$) there are in general eight independent kinematic covariants. For the elastic scattering ($A + B \to A + B$) their number is reduced to six due to time reversal invariance.For the 'superelastic' scattering of four identical particles ($A + A \to A + A$) the number of independent kinematic covariants is further reduced to five due to the symmetry under particle exchange.

The set of kinematic covariants is not unique. However, for the forthcoming it is essential that the $\mathcal{O}_i$ are chosen in such a way that the invariant amplitudes $c_i(s,t)$ do

not contain any kinematic singularities, but only 'physical' singularities demanded by unitarity. In the case of four spin-1/2 particles this condition is fulfilled by the set of eight covariants given in Ref. [21]. This set is based on the so-called Fermi-covariants S, P, V, A, T:

$$\begin{array}{lllllllllll} S & \equiv & \mathcal{O}_S & = & \mathbb{1}_4 \otimes \mathbb{1}_4 & , & T & \equiv & \mathcal{O}_T & = & \frac{1}{2}\sigma_{\mu\nu} \otimes \sigma^{\mu\nu} \quad , \\ P & \equiv & \mathcal{O}_P & = & \gamma_5 \otimes \gamma_5 & , & & & \mathcal{O}_6 & = & \mathbb{1}_4 \otimes \gamma_\mu P^\mu \; - \; \gamma_\mu Q^\mu \otimes \mathbb{1}_4 \quad , \\ V & \equiv & \mathcal{O}_V & = & \gamma_\mu \otimes \gamma^\mu & , & & & \mathcal{O}_7 & = & \gamma_5 \otimes \gamma_5 \gamma_\mu P^\mu \quad , \\ A & \equiv & \mathcal{O}_A & = & \gamma_5 \gamma_\mu \otimes \gamma_5 \gamma^\mu & , & & & \mathcal{O}_8 & = & \gamma_5 \gamma_\mu Q^\mu \otimes \gamma_5 \quad . \end{array} \tag{18}$$

These covariants are of the form $\mathcal{O}_i(P,Q) = \mathcal{O}_i^{(1)}(Q) \otimes \mathcal{O}_i^{(2)}(P)$ and the matrix elements have to be evaluated according to

$$\begin{aligned} &\overline{u}_C(\vec{p}_C,\lambda_C)\overline{u}_D(\vec{p}_D,\lambda_D)\mathcal{O}_i(P,Q)u_A(\vec{p}_A,\lambda_A)u_B(\vec{p}_B,\lambda_B) = \\ &\quad \left[\overline{u}_C(\vec{p}_C,\lambda_C)\mathcal{O}_i^{(1)}(Q)u_A(\vec{p}_A,\lambda_A)\right] \left[\overline{u}_D(\vec{p}_D,\lambda_D)\mathcal{O}_i^{(2)}(P)u_B(\vec{p}_B,\lambda_B)\right] . \end{aligned} \tag{19}$$

2.2 Dispersion relations for baryon-baryon amplitudes of $\pi\pi$ and $K\overline{K}$ exchange

For a general two-body scattering process the physical regions of the Mandelstam variables for the s-, t- and u-channel reaction are non-overlapping. Therefore the transition matrices in the three channels can be interpreted as independent branches of one operator $\mathcal{M}$ defined in the various kinematic regions. In the t-channel $(A + \overline{C} \to \overline{B} + D)$, for instance, the scattering amplitude is related to the invariant amplitudes by

$$\begin{aligned} &\langle\, \overline{B} - \vec{p}_B\lambda_B, D\vec{p}_D\lambda_D \,|\, T \,|\, A\vec{p}_A\lambda_A, \overline{C} - \vec{p}_C\lambda_C \,\rangle = \\ &\sum_{i=1}^{8} c_i(s,t) \left[\overline{v}_C(-\vec{p}_C,\lambda_C)\mathcal{O}_i^{(1)}(Q)u_A(\vec{p}_A,\lambda_A)\right] \left[\overline{u}_D(\vec{p}_D,\lambda_D)\mathcal{O}_i^{(2)}(P)v_B(-\vec{p}_B,\lambda_B)\right] . \end{aligned} \tag{20}$$

By introducing the concept of analyticity the invariant amplitudes in the three channels become closely related: The invariant amplitudes $c_i(s,t)$ are supposed to be analytic functions (except for the physical singularities) in the whole complex st-plane. Therefore, if all these physical singularities are known the $c_i(s,t)$ can be deduced at any point in the complex Mandelstam plane by the formulation of dispersion integrals.

The singularity structure of the invariant amplitudes is completely determined by the unitarity of the S-matrix. In terms of the scattering amplitude unitarity of the S-matrix is expressed as

$$i\left[\langle\, f \,|\, T \,|\, i \,\rangle - \langle\, f \,|\, T^\dagger \,|\, i \,\rangle\right] = \sum_n \frac{(2\pi)^4}{N_n} \delta^{(4)}(P_i - P_n) \langle\, f \,|\, T^\dagger \,|\, n \,\rangle \langle\, n \,|\, T \,|\, i \,\rangle \quad , \tag{21}$$

where P_n and $P_f = P_i$ denote the total 4-momenta. The summation in Eq. 21 is to be understood over all *physical* states n, i.e. states which are energetically accessible for the system with energy P_i^0.

The singularities of the baryon-baryon $(A + B \to C + D)$ amplitudes generated by (correlated) $\pi\pi$ and $K\overline{K}$ exchange are most easily derived using the unitarity relation

for the t-channel reaction $A+\overline{C}\to D+\overline{B}$. For this, one has to restrict the summation in Eq. 21 to physical $\pi\pi$ and $K\overline{K}$ states; i.e., contributions of single-meson poles or of heavier two-meson and multi-meson (e.g. 3π, 4π) channels are disregarded. In the c.m. system ($P_i = P_f = (\sqrt{t},\vec{0})$) the summation over the states n then reads

$$\begin{aligned}
&\sum_n \frac{(2\pi)^4}{N_n}\delta^{(4)}(P_i-P_n)\,|\,n\,\rangle\langle\,n\,| \;= \\
&\quad= \frac{1}{(2\pi)^2}\sum_{\mu\bar\mu=\pi\pi,K\bar K} N_{\mu\overline{\mu}}\int d^3k\,\delta(\sqrt{t}-2\omega_\mu(k))\frac{1}{4\omega_\mu(k)^2}\,|\,\mu\overline{\mu},\vec{k}\,\rangle\langle\,\mu\overline{\mu},\vec{k}\,| \\
&\quad= \frac{1}{32\pi^2}\sum_{\mu\bar\mu=\pi\pi,K\bar K} N_{\mu\overline{\mu}}\sqrt{\frac{t-4m_\mu^2}{t}}\,\theta(t-4m_\mu^2)\int d^2\hat{k}_{\mu\bar\mu}\,|\,\mu\overline{\mu},\vec{k}_{\mu\bar\mu}\,\rangle\langle\,\mu\overline{\mu},\vec{k}_{\mu\bar\mu}\,| \qquad (22)
\end{aligned}$$

with $\omega_\mu(k)=\sqrt{m_\mu^2+k^2}$ and the on-shell momentum $k_{\mu\bar\mu}=\sqrt{t/4-m_\mu^2}$. The symmetry factor $N_{\mu\overline{\mu}}$ is introduced in order to obtain the correct phase space in case of identical particles:

$$N_{\mu\bar\mu} = \begin{cases} 1/2 & \text{if} \quad \mu\overline{\mu}=\pi\pi \\ 1 & \text{if} \quad \mu\overline{\mu}=K\overline{K} \end{cases} \qquad (23)$$

Now, the kinematic covariants $\mathcal{O}_i$ in Eq. 18 have been chosen in such a way [21] that the left-hand side of Eq. 21 yields the imaginary part of the invariant amplitudes, i.e.

$$\begin{aligned}
&i\langle\, D\overline{B},\vec{q}\lambda_D\lambda_B\,|\,T-T^\dagger\,|\,A\overline{C},\vec{p}\lambda_A\lambda_C\,\rangle = \\
&-2\sum_i \mathit{Im}\,[c_i(s,t)]\,\overline{v}_C(-\vec{p},\lambda_C)\,\overline{u}_D(\vec{q},\lambda_D)\mathcal{O}_i(P,Q)u_A(\vec{p},\lambda_A)v_B(-\vec{q},\lambda_B)\,. \qquad (24)
\end{aligned}$$

Hence, the unitarity relation for the helicity amplitudes of the process $A+\overline{C}\to D+\overline{B}$ becomes in the c.m. system:

$$\begin{aligned}
&\sum_i \mathit{Im}\,[c_i(s,t)]\,\overline{v}_C(-\vec{p},\lambda_C)\,\overline{u}_D(\vec{q},\lambda_D)\mathcal{O}_i(P,Q)u_A(\vec{p},\lambda_A)v_B(-\vec{q},\lambda_B) = \\
&\quad= -\frac{1}{64\pi^2}\sum_{\mu\bar\mu=\pi\pi,K\bar K} N_{\mu\overline{\mu}}\sqrt{\frac{t-4m_\mu^2}{t}}\,\theta(t-4m_\mu^2) \\
&\qquad \int d^2\hat{k}_{\mu\bar\mu}\langle\,\mu\overline{\mu},\vec{k}_{\mu\bar\mu}\,|\,T\,|\,D\overline{B},\vec{q}\lambda_D\lambda_B\,\rangle^*\langle\,\mu\overline{\mu},\vec{k}_{\mu\bar\mu}\,|\,T\,|\,A\overline{C},\vec{p}\lambda_A\lambda_C\,\rangle \quad . \qquad (25)
\end{aligned}$$

From the beginning, the unitarity relations 21, 25 are just defined above the kinematic threshold of the process $A+\overline{C}\to D+\overline{B}$, that is for $t\geq t_0\equiv\max\{M_A+M_C, M_B+M_D\}$. However, they can be continued analytically into the pseudophysical region ($4m_\pi^2\leq t\leq t_0$) as discussed in Ref. [22]. Below the kinematic threshold the baryon-antibaryon momenta become imaginary. According to Ref. [22] the unitarity relation for the S-matrix, which can be written symbolically as $[S(p)]^*\,S(p)=1$, has to be continued to complex momenta by $[S(p^*)]^*\,S(p)=1$. As explained in Ref. [23], for imaginary momenta this is equivalent to evaluating the expression $[S(p)]^*\,S(p)=1$ with real dummy variables for the momenta and replacing these dummy variables at the very end (after having performed all complex conjugations) by the imaginary momenta.

The right-hand side of Eq. 25 obviously vanishes below the $\pi\pi$ threshold at $t=4m_\pi^2$. Since for the processes considered here the left-hand cuts, which are due to unitarity

constraints for the u-channel process, do not extend up to $t = 4m_\pi^2$ (for fixed s lying inside the physical s-channel region), the invariant amplitudes $c_i(s,t)$ are real-analytic functions of t, i.e. $c_i(s,t^*) = c_i(s,t)^*$. According to Eq. 25 and

$$2iIm\,[c_i(s,t+i\epsilon)] = c_i(s,t+i\epsilon) - c_i(s,t-i\epsilon) \quad , \tag{26}$$

$c_i(s,t)$ has a branch cut along the real t-axis extending from $t = 4m_\pi^2$ to $t = +\infty$. Corresponding statements hold for the $K\overline{K}$ branch cut.

For c.m. energies below $1\,GeV$ the $\pi\pi$ interaction is dominated by the $JI = 00, 11$ partial waves [13]. At low transfered momenta, relevant in low-energy baryon-baryon scattering, correlations between two exchanged pions (kaons) are therefore only considerable when the exchanged $\pi\pi$ ($K\overline{K}$) system is in a state with relative angular momentum $J = 0$ and isospin $I = 0$ ('σ-channel') or $J = 1$ and $I = 1$ ('ρ-channel') [12]. In our approach only the correlated part of two-pion and two-kaon exchange is evaluated by dispersiontheoretic means. The uncorrelated part has to be calculated directly in the s-channel in order to include all t-channel partial waves and to guarantee the correct energy dependence of this contribution. In the ΛN channel, for instance, the iterative two-pion exchange with an $N\Sigma$ intermediate state becomes complex above the $N\Sigma$ threshold. This behavior cannot be reproduced with a single-variable dispersion relation in the t-channel which will be applied to the correlated contribution (see below).

Hence, the following dispersiontheoretic considerations can be limited to the σ and ρ channel of $\pi\pi$ ($K\overline{K}$) exchange. For this, the $B\overline{B'} \to \mu\overline{\mu}$ amplitudes on the right-hand side of the unitarity relation 25 are decomposed into partial waves [24]. Choosing the coordinate system so that $\vec{p}$ points along the $\hat{z}$-axis and $\vec{q}$ lies in the $\hat{x}\hat{z}$-plane the partial wave decomposition gives

$$\begin{aligned}&\int d^2\hat{k}_{\mu\bar{\mu}} \langle\, \mu\overline{\mu}, \vec{k}_{\mu\bar{\mu}} \,|\, T \,|\, D\overline{B}, \vec{q}\,\lambda_D\lambda_B \,\rangle^* \langle\, \mu\overline{\mu}, \vec{k}_{\mu\bar{\mu}} \,|\, T \,|\, A\overline{C}, \vec{p}\,\lambda_A\lambda_C \,\rangle = \\ &\sum_J \frac{2J+1}{4\pi} d^J_{\lambda_A-\lambda_C,\lambda_D-\lambda_B}(\cos\vartheta_t) \langle\, \mu\overline{\mu} \,|\, T^J(t) \,|\, D\overline{B}, \lambda_D\lambda_B \,\rangle^* \langle\, \mu\overline{\mu} \,|\, T^J(t) \,|\, A\overline{C}, \lambda_A\lambda_C \,\rangle,\end{aligned} \tag{27}$$

where the on-shell momenta $p = p(t)$ and $q = q(t)$ (see Eq. 8) are suppressed as arguments of the partial wave decomposed T matrix elements. Note that the right-hand side depends on the Mandelstam variable s only via the angle $\vartheta_t = \vartheta_t(s,t)$ (see Eq. 9) between $\vec{p}$ and $\vec{q}$. Now, by restricting the sum over J in Eq. 27 to $J = 0$ ($J = 1$) the contribution $c_i^{(J)}(s,t) = c_i^\sigma(s,t)$ ($c_i^\rho(s,t)$) of the $\pi\pi$ and $K\overline{K}$ intermediate states to the discontinuity of the invariant amplitudes $c_i(s,t)$ in the σ (ρ) channel can be isolated. From the unitarity relation 25 we obtain

$$\begin{aligned}&\sum_i Im\left[c_i^{(J)}(s,t)\right] \overline{v}_C(-\vec{p},\lambda_C)\, \overline{u}_D(\vec{q},\lambda_D) \mathcal{O}_i(P,Q) u_A(\vec{p},\lambda_A) v_B(-\vec{q},\lambda_B) = \\ &\quad = \sum_{\mu\bar{\mu}=\pi\pi,K\bar{K}} d^J_{\lambda_A-\lambda_C,\lambda_D-\lambda_B}(\cos\vartheta_t) H^J_{\mu\overline{\mu}}(t) \\ &\qquad\qquad \langle\, \mu\overline{\mu} \,|\, T^J(t) \,|\, D\overline{B}, \lambda_D\lambda_B \,\rangle^* \langle\, \mu\overline{\mu} \,|\, T^J(t) \,|\, A\overline{C}, \lambda_A\lambda_C \,\rangle \quad ,\end{aligned} \tag{28}$$

with the abbreviation

$$H^J_{\mu\overline{\mu}}(t) \equiv -\frac{2J+1}{256\pi^3} N_{\mu\overline{\mu}} \sqrt{\frac{t-4m_\mu^2}{t}}\, \theta(t-4m_\mu^2) \quad . \tag{29}$$

Eq. 28 is a system of linear equations for the discontinuities $Im\left[c_i^{(J)}(s,t)\right]$. Its solution provides the discontinuities as linear combinations of the following products of $B\overline{B'} \rightarrow \mu\overline{\mu}$ helicity amplitudes:

$$F^J_{\lambda_D\lambda_B,\lambda_A\lambda_C} \equiv \sum_{\mu\bar{\mu}=\pi\pi,K\bar{K}} H^J_{\mu\overline{\mu}}(t)\langle\, \mu\overline{\mu}\,|\, T^J(t)\,|\, D\overline{B}, \lambda_D\lambda_B\,\rangle^* \,\langle\, \mu\overline{\mu}\,|\, T^J(t)\,|\, A\overline{C}, \lambda_A\lambda_C\,\rangle \quad . \tag{30}$$

From the symmetry properties of the $B\overline{B'} \rightarrow \mu\overline{\mu}$ helicity amplitudes, which are due to the parity invariance of the underlying strong interaction (see e.g. Ref. [20]),

$$\langle\, \mu\overline{\mu}\,|\, T^J(k,q)\,|\, B\overline{B'}, \lambda\lambda'\,\rangle = \langle\, \mu\overline{\mu}\,|\, T^J(k,q)\,|\, B\overline{B'}, -\lambda-\lambda'\,\rangle \quad , \tag{31}$$

the following relations can be deduced:

$$F^J_{\lambda_D\lambda_B,\lambda_A\lambda_C} = F^J_{(-\lambda_D)(-\lambda_B),\lambda_A\lambda_C} = F^J_{\lambda_D\lambda_B,(-\lambda_A)(-\lambda_C)} = F^J_{(-\lambda_D)(-\lambda_B),(-\lambda_A)(-\lambda_C)} \quad , \tag{32}$$

Therefore only four linear independent $F^J_{\lambda_D\lambda_B,\lambda_A\lambda_C}$ exist for $J>0$. For $J=0$ there is only one independent $B\overline{B'} \rightarrow \mu\overline{\mu}$ helicity amplitude, $\langle\, \mu\overline{\mu}\,|\, V^J(k,q)\,|\, B\overline{B'}, ++\,\rangle$, and consequently only one independent $F^J_{\lambda_D\lambda_B,\lambda_A\lambda_C}$. Using the explicit representation of helicity spinors in Appendix A the evaluation of the matrix elements of the kinematic covariants $\mathcal{O}_i$ in Eq. 28 is straightforward.

In case of unequal baryon masses $M_B \neq M_{B'}$ the analytic structure of the $B\overline{B'} \rightarrow \mu\overline{\mu}$ helicity amplitudes is much more involved compared to the case when the baryon masses are equal. This complicates an analytic continuation of the amplitudes to the pseudophysical region. The $\Lambda\overline{\Sigma}$ channel is the only channel where this problem arises. In order to facilitate an easy handling of the expressions we treat this channel approximately by setting the mass of the Λ and of the Σ equal to the average mass $(M_\Lambda + M_\Sigma)/2$. This approximation is justified for two reasons: First the mass difference between Λ and Σ hyperon is small ($77MeV$) on the baryonic scale. Second, since we evaluate the uncorrelated part directly in the s-channel the important energy dependence of this contribution is not affected by our approximation. The correlated contribution on the other hand is known to have a rather smooth energy dependence which is supposed not to change drastically by our approximation.

In the following we restrict ourselves therefore to the case where

$$M_A = M_C \equiv M \quad , \qquad M_B = M_D \equiv M' \quad . \tag{33}$$

Solving the system of linear equations 28 yields that in the σ channel ($J=0$) all discontinuities $Im\,[c_i^\sigma(s,t)]$ vanish except for the scalar component

$$\begin{aligned} &Im\,[c_S^\sigma(t)] = \frac{M'M}{qp} F^{J=0}_{++,++} \quad , \\ &Im\,[c_P^\sigma(s,t)] = Im\,[c_V^\sigma(s,t)] = Im\,[c_A^\sigma(s,t)] = Im\,[c_T^\sigma(s,t)] = Im\,[c_6^\sigma(s,t)] = 0 \quad , \end{aligned} \tag{34}$$

In contrast, in the ρ channel only the axial-vector component vanishes:

$$\begin{aligned}
Im\,[c_S^\rho(s,t)] &= -\tfrac{1}{2}g(t)\sqrt{t}qp\cos\vartheta_t(s,t) \quad [\quad -2\sqrt{t}(M'+M)(2F^{J=1}_{++,++}+F^{J=1}_{+-,+-}) \\
&\qquad\qquad +\sqrt{2}(t+4M'M)(F^{J=1}_{++,+-}+F^{J=1}_{+-,++})], \\
Im\,[c_P^\rho(s,t)] &= g(t)qp\cos\vartheta_t(s,t) \quad [\quad (M'+M)(t-4M'M)F^{J=1}_{+-,+-} \\
&\qquad\qquad -2\sqrt{2}\sqrt{t}(p^2F^{J=1}_{++,+-}+q^2F^{J=1}_{+-,++})] \quad , \\
Im\,[c_V^\rho(t)] &= g(t)\sqrt{t} \quad [\quad -\sqrt{t}(M'+M)(t/4-M'^2-M^2+M'M)F^{J=1}_{+-,+-} \\
&\qquad\qquad +2\sqrt{2}M'M(p^2F^{J=1}_{++,+-}+q^2F^{J=1}_{+-,++})] \quad , \\
Im\,[c_A^\rho(s,t)] &= 0 \quad , \\
Im\,[c_T^\rho(t)] &= \tfrac{1}{4}g(t)t \quad [\quad (M'+M)(t-4M'M)F^{J=1}_{+-,+-} \\
&\qquad\qquad -2\sqrt{2}\sqrt{t}(p^2F^{J=1}_{++,+-}+q^2F^{J=1}_{+-,++})] \quad , \\
Im\,[c_6^\rho(t)] &= g(t)\sqrt{t} \quad [\quad (M'^2-M^2)\sqrt{t}F^{J=1}_{+-,+-} \\
&\qquad\qquad -2\sqrt{2}(p^2M'F^{J=1}_{++,+-}-q^2MF^{J=1}_{+-,++})] \quad ,
\end{aligned} \tag{35}$$

with

$$g(t) \equiv \frac{M'M}{2q^2p^2(M'+M)t} \quad . \tag{36}$$

The discontinuity of the pseudoscalar and the tensor component are obviously related by

$$Im\,[c_P^\rho(s,t)] = \frac{4qp\cos\vartheta_t}{t} Im\,[c_T^\rho(t)] \overset{(9)}{=} \frac{u-s}{t} Im\,[c_T^\rho(t)] \quad . \tag{37}$$

Consequently only four of the five nonvanishing discontinuities are linear independent in alignment with the number of independent $F^{J=1}_{\lambda_D\lambda_B,\lambda_A\lambda_C}$.

Except for poles (corresponding to single-particle exchange) and cuts the invariant amplitudes $c_i^{(J)}(s\ \text{fixed}, t)$ are real-analytic functions of t. Therefore, fixed-s dispersion relations can be formulated for the $c_i^{(J)}(s,t)$. Since we want to restrict the dispersion-theoretic evaluation to the contribution of (correlated) $\pi\pi$ and $K\overline{K}$ exchange to the baryon-baryon amplitudes we take into account only those singularities which are generated by $\pi\pi$ and $K\overline{K}$ intermediate states, namely the discontinuities of Eqs. 34–35 due to the $\pi\pi$ and $K\overline{K}$ unitarity cut ('right-hand cut'). The left-hand cuts, which are due to unitarity constraints for the u-channel reaction, can be neglected in the baryon-baryon channels considered here, since they start at large, negative t-values (from which they extend to $-\infty$) and are therefore far away from the physical region relevant for low-energy s-channel processes. For identical baryons (e.g. $NN \to NN$) this is only true if the dispersion relations are applied only to the direct baryon-baryon amplitude and the antisymmetrization of the amplitudes is not taken into account from the very beginning but just for the final s-channel amplitudes. Otherwise, crossing of the exchange diagram would result in a u-channel cut starting at $u = 4m_\pi^2$ which could not be neglected in the dispersion integrals [16].

In this work the $B\overline{B'} \to \mu\overline{\mu}$ amplitudes, which enter in Eq. 28, are derived from a microscopic model which is based on the hadron-exchange picture (see Sect. 2.4). Of course, this model has a limited range of validity: for energies far beyond $t'_{max} \approx 100m_\pi^2$

it cannot provide reliable results. The dispersion integral for the invariant amplitudes extending in principle along the whole $\pi\pi$ right-hand cut has therefore to be limited to an upper bound (t'_{max}). In addition left-hand cuts and unphysical cuts introduced for instance by the form factor prescription of the microscopic model for the $B\overline{B'} \to \mu\overline{\mu}$ amplitudes are neglected. Because of these approximations of the exact expressions, which are necessary in order to obtain a solution of the physical problem, the formulation of either a dispersion relation or a subtracted dispersion relation might lead to different results for the amplitudes although both should be mathematically equivalent. However, the ambiguity which dispersion relation to choose can be avoided by demanding that the analytic structure of the resulting $c_i^{(J)}(s,t)$ should agree as far as possible with the expressions for sharp σ and ρ exchange in the baryon-baryon interaction.

The transition amplitude $\mathcal{M}^\sigma$ for the exchange of a scalar σ meson with mass m_σ between two $J^P = 1/2^+$ baryons A and B follows from the interaction Lagrangians ($X = A, B$)

$$\mathcal{L}_{XX\sigma}(x) = g_{XX\sigma}\overline{\psi}_X(x)\psi_X(x)\phi_\sigma(x) \tag{38}$$

(See Appendix A for the hadronic field operators.) The result is

$$\mathcal{M}^\sigma(t) = g_{AA\sigma}g_{BB\sigma}\frac{F_\sigma^2(t)}{t-m_\sigma^2}\mathcal{O}_S \quad , \tag{39}$$

where a form factor $F_\sigma(t)$ has to be applied at each vertex since the exchanged σ meson is far away from its mass-shell. This form factor is parametrized in the conventional monopole form

$$F_\sigma(t) = \frac{\Lambda_\sigma^2 - m_\sigma^2}{\Lambda_\sigma^2 - t} \quad . \tag{40}$$

with a cutoff mass Λ_σ assumed to be uniform for both vertices.

The construction of the amplitude for ρ exchange in the transition $A + B \to C + D$ (with $M_A = M_C \equiv M$ and $M_B = M_D \equiv M'$) starts from the interaction Lagrangians

$$\begin{aligned}\mathcal{L}_{XY\rho}(x) = \quad & g_{XY\rho}\overline{\psi}_X(x)\gamma_\mu\psi_Y(x)\phi_\rho^\mu(x) \\ + \quad & \frac{f_{XY\rho}}{4M_N}\overline{\psi}_X(x)\sigma_{\mu\nu}\psi_Y(x)(\partial^\mu\phi_\rho^\nu(x) - \partial^\nu\phi_\rho^\mu(x)) \\ & (+\text{h.c., if } X \neq Y)\end{aligned} \tag{41}$$

with $(XY) = (AC), (BD)$. According to the conventional Feynman rules, using Eq. 11 and the generalized Gordon decomposition [25] for the spinors of two Dirac particles X and Y,

$$\overline{u}_X(\vec{p}\,',\lambda')\left[(M_X+M_Y)\gamma^\mu - (p'+p)^\mu - i\sigma^{\mu\nu}(p'-p)_\nu\right]u_Y(\vec{p},\lambda) = 0 \quad , \tag{42}$$

the transition amplitude $\mathcal{M}^\rho$ comes out to be

$$\begin{aligned}\mathcal{M}^\rho(P,Q) = \frac{-1}{t-m_\rho^2} \quad \Big[\quad & \frac{f_{AC\rho}f_{BD\rho}}{4M_N^2}(s-u)\ \mathcal{O}_S \\ & + G_{AC\rho}G_{BD\rho}\ \mathcal{O}_V \\ & + \frac{G_{AC\rho}f_{BD\rho} - G_{BD\rho}f_{AC\rho}}{2M_N}\ \mathcal{O}_6 \\ & + \frac{G_{AC\rho}f_{BD\rho} + G_{BD\rho}f_{AC\rho}}{2M_N}\ \mathcal{P}_2 \Big] \quad ,\end{aligned} \tag{43}$$

with

$$G_{XY\rho} \equiv g_{XY\rho} + \frac{M_X + M_Y}{2M_N} f_{XY\rho} \tag{44}$$

and the 4-momenta P and Q defined in Eq. 10. $\mathcal{P}_2$ is one of the so-called perturbative covariants introduced in Ref. [15] as

$$\mathcal{P}_2 = -\mathbb{1}_4 \otimes \gamma_\mu P^\mu \; - \; \gamma_\mu Q^\mu \otimes \mathbb{1}_4 \quad . \tag{45}$$

$\mathcal{P}_2$ can be expanded in terms of the $\mathcal{O}_i$ of Eq. 18 since they form a complete set of kinematic covariants. For the given baryon masses one finds

$$\mathcal{P}_2 = \frac{1}{2(M+M')} \left[(u-s)(\mathcal{O}_S + \mathcal{O}_P) - 4MM'\mathcal{O}_V + t\mathcal{O}_T + 2(M-M')\mathcal{O}_6 \right] \quad . \tag{46}$$

After replacing $\mathcal{P}_2$ in Eq. 43 the final result for $\mathcal{M}^\rho$ reads

$$\begin{aligned}
\mathcal{M}^\rho(P,Q) = \frac{-F_\rho^2(t)}{t-m_\rho^2} \quad \Bigg\{ \quad & \frac{g_{AC\rho} f_{BD\rho} + f_{AC\rho} g_{BD\rho}}{4M_N M_{tot}} (u-s)\, \mathcal{O}_S \\
& \left[g_{AC\rho} g_{BD\rho} + g_{AC\rho} f_{BD\rho} \frac{M'^2}{M_N M_{tot}} + f_{AC\rho} g_{BD\rho} \frac{M^2}{M_N M_{tot}} \right] \mathcal{O}_V \\
& \left(g_{AC\rho} f_{BD\rho} \frac{M'}{M_N M_{tot}} - f_{AC\rho} g_{BD\rho} \frac{M_A}{M_N M_{tot}} \right) \mathcal{O}_6 \\
& \frac{G_{AC\rho} f_{BD\rho} + f_{AC\rho} G_{BD\rho}}{4M_N M_{tot}} \left[(u-s)\, \mathcal{O}_P \; + t\, \mathcal{O}_T \, \right] \Bigg\}
\end{aligned} \tag{47}$$

with $M_{tot} = M + M'$ and the form factor $F_\rho(t)$ parametrized according to Eq. 40.

By comparison of the discontinuities in the σ and ρ channel in Eqs. 34, 35 with the transition amplitudes for sharp σ and ρ exchange it follows that c_S^σ, c_V^ρ and c_6^ρ obey an unsubtracted dispersion relation,

$$c_i^{(J)}(s,t) = \frac{1}{\pi} \int_{4m_\pi^2}^{t'_{max}} \frac{Im\left[c_i^{(J)}(s,t')\right]}{t'-t} dt' \quad . \tag{48}$$

The tensor component of sharp ρ exchange is proportional to t (cf. 47). In order to generate this factor t also for the tensor component of correlated $\pi\pi$ and $K\overline{K}$ exchange a subtracted dispersion relation (subtraction point t_0 and subtraction constant $c_T^\rho(s,t_0) = 0$) is assumed for the invariant amplitude $c_T^\rho(s,t)$:

$$c_T^\rho(s,t) = \frac{t}{\pi} \int_{4m_\pi^2}^{t'_{max}} \frac{Im\left[c_T^\rho(s,t')\right]/t'}{t'-t} dt' \quad . \tag{49}$$

Similarly, the $u-s$ dependence of the (pseudo-)scalar component of sharp ρ exchange (cf. Eq. 47) can be reproduced by assuming a subtracted dispersion relation for $c_S^\rho(s,t)$ and $c_P^\rho(s,t)$ with $t_0(s) = \Sigma - 2s$ and $c_{S,P}^\rho(s,t_0(s)) = 0$:

$$c_{S,P}^\rho(s,t) = \frac{u-s}{\pi} \int_{4m_\pi^2}^{t'_{max}} \frac{Im\left[c_{S,P}^\rho(s,t')\right]/(u'-s)}{t'-t} dt' \quad , \tag{50}$$

where according to Eq. 2 $s+t+u = s+t'+u' = \Sigma$.

2.3 Baryon-baryon interaction arising from correlated $\pi\pi$ and $K\overline{K}$ exchange

The invariant amplitudes constructed in the preceding Section using dispersion theory still contain the uncorrelated contributions of $\pi\pi$ and $K\overline{K}$ exchange. Investigating the problem of baryon-baryon scattering requires the knowledge of the on-shell scattering amplitude T, which is usually obtained as a solution of a scattering equation $T = V + VGT$ that iterates the interaction kernel V. In general, V contains besides other terms one-pion and one-kaon exchange contributions as well as the contribution $V_{2\pi}$ from two-pion and two-kaon exchange. But iterating π and K exchange in the second order term VGV also generates two-pion and two-kaon exchange contributions to the scattering amplitude. In order to avoid double counting these 'iterative' contributions therefore have to be left out from the dispersiontheoretically calculated $V_{2\pi}$. As stated above we even go beyond this and subtract all uncorrelated contributions from $V_{2\pi}$. By this the dispersiontheoretic calculations can be restricted to the σ and ρ channel (since only there significant correlations occur in the kinematic region considered), whereas the uncorrelated contributions are evaluated in the s-channel and therefore contain all t-channel partial waves.

In order to eliminate the uncorrelated contributions from $V_{2\pi}$ we determine the discontinuities $Im[c^{(J)}_{i,Born}(s,t)]$ generated from the $B\overline{B'} \to \mu\overline{\mu}$ Born amplitudes V^J (i.e., no $\pi\pi - K\overline{K}$ correlations included) using as before the unitarity relation 28 (with T^J replaced by V^J) and subtract them finally from the full discontinuities $Im[c^{(J)}_i(s,t)]$. (The contributions of the ρ-pole diagram to the $B\overline{B'} \to \mu\overline{\mu}$ Born amplitudes must not be subtracted since the corresponding s-channel processes are not included explicitly in V.) Hence, for the invariant amplitudes of *correlated* $\pi\pi$ and $K\overline{K}$ exchange, $\tilde{c}^{(J)}_i(s,t)$, the (unsubtracted) dispersion relation 48 has to be modified to

$$\tilde{c}^{(J)}_i(s,t) = \frac{1}{\pi}\int_{4m_\pi^2}^{t'_{max}} \frac{\rho^{(J)}_i(s,t')}{t'-t}dt' \quad , \qquad (J) = \sigma, \rho \quad , \tag{51}$$

where the spectral function $\rho^{(J)}_i$ is given by

$$\rho^{(J)}_i(s,t') \equiv Im\left[c^{(J)}_i(s,t')\right] - Im\left[c^{(J)}_{i,Born}(s,t')\right] \quad . \tag{52}$$

Corresponding expressions hold for the subtracted dispersion relations 49 and 50.

Now the baryon-baryon helicity amplitudes arising from correlated $\pi\pi$ and $K\overline{K}$ exchange can be evaluated according to Eqs. 16, 17

$$\begin{aligned}\langle\, CD, \vec{q}\lambda_C\lambda_D \,|\, V^{(J)}_{2\pi} \,|\, AB, \vec{p}\lambda_A\lambda_B \,\rangle = \\ \overline{u}_C(\vec{q},\lambda_C)\,\overline{u}_D(-\vec{q},\lambda_D)\,\mathcal{V}^{(J)}_{2\pi}(P,Q)\,u_A(\vec{p},\lambda_A)\,u_B(-\vec{p},\lambda_B)\end{aligned}$$

with

$$\mathcal{V}^{(J)}_{2\pi}(P,Q) \equiv \sum_i \tilde{c}^{(J)}_i(s,t)\mathcal{O}_i(P,Q) \qquad ((J) = \sigma, \rho) \quad . \tag{53}$$

The partial wave decomposition of these matrix elements then proceeds as usual (see e.g. Ref. [3]).

Of course, when iterating the baryon-baryon interaction kernel in a scattering equation $V_{2\pi,corr}$ has to be known off-shell. However, dispersion theory applies only to

on-shell amplitudes and does not provide any information on the off-shell behavior of the amplitudes. Therefore an arbitrary prescription for the off-shell extrapolation of $V_{2\pi,corr}$ has to be defined [12], which is certainly a drawback of the dispersiontheoretical derivation of this potential. Nevertheless the characteristic features of correlated $\pi\pi$ and $K\overline{K}$ exchange like the strength of $V_{2\pi,corr}$ in the various baryon-baryon channels can already be discussed by means of the unique on-shell amplitudes. Therefore we postpone the discussion of how to extrapolate $V_{2\pi,corr}$ off-shell to a subsequent work.

The dispersiontheoretic amplitudes for correlated $\pi\pi$ and $K\overline{K}$ exchange (Eqs. 34, 35) have been constructed in such a way that their operator structure agrees as far as possible with sharp σ and ρ exchange 39, 47. Therefore our results for the correlated exchange can be parametrized in terms of σ and ρ exchange; i.e., the products of coupling constants for σ and ρ exchange are replaced by effective coupling strengths $G^{(J)}(s,t)$, which contain the full s- and t-dependence of the dispersiontheoretic results. In the σ channel this gives for the elastic baryon-baryon process $A+B\rightarrow A+B$

$$g_{AA\sigma}g_{BB\sigma} \quad \longrightarrow \quad G^{\sigma}_{AB\rightarrow AB}(t) = \frac{(t-m_\sigma^2)}{F_\sigma^2(t)}\tilde{c}^{\sigma}_{S}(t) = \frac{(t-m_\sigma^2)}{\pi F_\sigma^2(t)}\int_{4m_\pi^2}^{t'_{max}}\frac{\rho^\sigma_S(t')}{t'-t}dt' \tag{54}$$

Note that sharp σ exchange (Eq. 39) would correspond to a spectral function

$$\rho^\sigma_S(s,t') = -g_{AA\sigma}g_{BB\sigma}\delta(t'-m_\sigma^2) \tag{55}$$

(except for form factors). This suggests to interpret the spectral function as a function that denotes the strength of an exchange process depending on the invariant mass of the exchanged system (here: $\pi\pi$, $K\overline{K}$).

By comparing the coefficients of the kinematic covariants $\mathcal{O}_i$ in Eqs. 47 and 53 we obtain in the ρ-channel:

$$\begin{aligned}
g_{AB\rho}g_{BD\rho} &\rightarrow {}^{VV}G^{\rho}_{AB\rightarrow CD}(t) = \frac{(t-m_\rho^2)}{F_\rho^2(t)}\left[4MM'\frac{\tilde{c}^\rho_S(s,t)}{u-s} - \tilde{c}^\rho_V(t) + (M'-M)\tilde{c}^\rho_6(t)\right], \\
g_{AB\rho}f_{BD\rho} &\rightarrow {}^{VT}G^{\rho}_{AB\rightarrow CD}(t) = -\frac{(t-m_\rho^2)}{F_\rho^2(t)}M_N\left[4M\frac{\tilde{c}^\rho_S(s,t)}{u-s} + \tilde{c}^\rho_6(t)\right], \\
f_{AB\rho}g_{BD\rho} &\rightarrow {}^{TV}G^{\rho}_{AB\rightarrow CD}(t) = -\frac{(t-m_\rho^2)}{F_\rho^2(t)}M_N\left[4M'\frac{\tilde{c}^\rho_S(s,t)}{u-s} - \tilde{c}^\rho_6(t)\right], \\
f_{AB\rho}f_{BD\rho} &\rightarrow {}^{TT}G^{\rho}_{AB\rightarrow CD}(t) = \frac{(t-m_\rho^2)}{F_\rho^2(t)}4M_N^2\frac{\tilde{c}^\rho_S(s,t)-\tilde{c}^\rho_P(s,t)}{u-s}.
\end{aligned} \tag{56}$$

Obviously the effective coupling strengths do not depend on s but only on t. This is only possible by choosing the subtracted dispersion relation 50 for $\tilde{c}^\rho_{S,P}(s,t)$, since the integrand of the dispersion integral becomes independent of s ($Im[c^\rho_{S,P}(s,t')] \propto \cos\vartheta_t(s,t') \propto u'-s$). Therefore $\tilde{c}^\rho_{S,P}(s,t)$ depends on s only by the factor $(u-s)$, which cancels out exactly when calculating the effective coupling strengths. It should be emphasized that the parametrization of $V_{2\pi,corr}$ discussed here does (so far) not contain any approximations.

2.3.1 Isospin-Crossing

Up to now, in favor of a clear representation of the dispersiontheoretical calculation, we have suppressed isospin degrees of freedom. The isospin structure of the $B\overline{B'}\rightarrow\mu\overline{\mu}$

amplitudes will be discussed in the next Sections when the microscopic model for these amplitudes is presented. However, when 'crossing' is applied to the baryon-baryon (s-channel) and baryon-antibaryon (t-channel) amplitudes it has to be kept in mind that the total isospin in the various channels is constructed from different combinations of particle isospins. The total isospin I_s of the s-channel process $A+B \rightarrow C+D$ is composed out of $[I_A \otimes I_B]_{I_s}$ or $[I_C \otimes I_D]_{I_s}$ and that of the t-channel process $A+\overline{C} \rightarrow D+\overline{B}$ out of $[I_A \otimes I_{\bar{C}}]_{I_t}$ or $[I_D \otimes I_{\bar{B}}]_{I_t}$.

Consequently, besides the analytic continuation of the invariant amplitudes in s and t the recoupling of the particle isospins has to be taken into account when crossing amplitudes. Therefore, the isospin amplitudes $T_s^{AB\rightarrow CD}(I_s)$ and $T_t^{A\overline{C}\rightarrow D\overline{B}}(I_t)$ of the s- and t-channel processes being independent of the isospin projections m_s and m_t are linearly related:

$$T_s^{AB\rightarrow CD}(I_s) = \sum_{I_t} X_{AB,CD}(I_s,I_t) T_t^{A\overline{C}\rightarrow D\overline{B}}(I_t) \quad , \tag{57}$$

where $X_{AB,CD}(I_s,I_t)$ is the so-called isospin-crossing matrix. Note that our isospin-crossing matrix differs from the $\tilde{X}_{AB,CD}(I_s,I_t)$ introduced in Refs. [26, 27] since their t-channel process ($\overline{D}+B \rightarrow C+\overline{A}$) differs from the one in Sect. 2.1:

$$X_{AB,CD}(I_s,I_t) = (-1)^{I_B+I_D+I_t-2I_s} \tilde{X}_{BA,DC}(I_s,I_t) \quad . \tag{58}$$

As shown in Ref. [26] the isospin-crossing matrix $\tilde{X}_{AB,CD}(I_s,I_t)$ can be expressed in terms of Clebsch-Gordan coefficients

$$\begin{aligned}\tilde{X}_{AB,CD}(I_s,I_t) =& \\ &\eta_A\eta_D \sum_{\substack{m_A,m_B,\\ m_C,m_D,m_t}} (-1)^{I_A+I_D+m_A+m_D} \langle\, I_A I_B m_A m_B \,|\, I_s m_s \,\rangle \langle\, I_C I_D m_C m_D \,|\, I_s m_s \,\rangle \\ &\qquad \langle\, I_C I_A m_C(-m_A) \,|\, I_t m_t \,\rangle \langle\, I_D I_B (-m_D) m_B \,|\, I_t m_t \,\rangle\end{aligned} \tag{59}$$

with m_s ($|m_s| \leq I_s$) being arbitrary. For a particle A with isospin I_A and isospin projection m_A the particle state $|\,A\,\rangle$ and the isospin state $|\,Im\,\rangle_A$ might differ in sign. The isospin state of the antiparticle $\overline{A}$ is generated by applying the G-parity operator $\mathcal{G}$ to $|\,Im\,\rangle_A$ [26]:

$$|\,Im\,\rangle_{\bar{A}} = \eta_A \mathcal{G}\,|\,Im\,\rangle_A \quad . \tag{60}$$

With the phase convention used here for the $SU(3)$ field operators (e.g. when calculating the isospin factors of the $B\overline{B'} \rightarrow \mu\overline{\mu}$ amplitudes in the next Section) the phase η_A, which is independent of m_A, comes out to be [27]

$$\eta_A = (-1)^{I_A - Y_A/2} \quad , \tag{61}$$

where Y denotes the hypercharge of particle A. Note that this phase convention differs from the one used in Ref. [26]. For the baryon-baryon processes considered in this work the isospin-crossing matrices are tabulated in Tab. 1.

By the partial wave decomposition the amplitude of correlated $\pi\pi$ and $K\overline{K}$ exchange is separated into the contributions of the σ- ($I_t = 0$) and ρ-channel ($I_t = 1$). Suppressing the spin-momentum dependence the isospin amplitude can be written as

$$T_s(I_s) = X(I_s,0)T_t^\sigma + X(I_s,1)T_t^\rho \quad . \tag{62}$$

The column $X(I_s,0)$ $(X(I_s,1))$ of the isospin-crossing matrix agrees except for a constant factor F_σ (F_ρ) with the isospin factors for t-channel exchange of a σ (ρ) meson in the corresponding s-channel process. Conventionally these constant factors F_σ and F_ρ, which are also tabulated in Tab. 1, are extracted from the isospin-crossing matrix and put into the spectral functions 52, so that the isospin factors for the s-channel potential $V_{2\pi}$ agree with the isospin factors of σ and ρ exchange.

2.4 A microscopic model for the $B\overline{B'} \to \pi\pi, K\overline{K}$ transition amplitudes

In the preceding Sections we have outlined the dispersiontheoretic calculation of correlated $\pi\pi$ and $K\overline{K}$ exchange in the baryon-baryon interaction starting from amplitudes for the transition of a baryon-antibaryon ($B\overline{B'}$) state to two pions ($\pi\pi$) or a kaon and a antikaon ($K\overline{K}$). These amplitudes have to be known in the so-called pseudophysical region, i.e. for energies below the $B\overline{B'}$ threshold. However, in case of the process $N\overline{N} \to \pi\pi$, these amplitudes can be derived from empirical data by analytic continuation of πN and $\pi\pi$ scattering amplitudes, which are extracted from scattering data, into the pseudophysical region [18, 28, 29, 30]. Corresponding analyses for the transitions $Y\overline{Y'} \to \pi\pi, K\overline{K}$ are out of sight since the required empirical information (e.g. $\pi\Lambda$ scattering data) does not exist. An evaluation of correlated $\pi\pi$ and $K\overline{K}$ exchange in the YN or YY interaction therefore necessitates the construction of a microscopic model for the $B\overline{B'} \to \pi\pi, K\overline{K}$ amplitudes. This model can be tested against the quasiempirical information for the $N\overline{N} \to \pi\pi$ amplitudes and then has to be extrapolated to the other channels of interest. In addition, only the use of a microscopic model for the $B\overline{B'} \to \pi\pi, K\overline{K}$ amplitudes allows a consistent treatment of medium modifications of the baryon-baryon interaction.

The microscopic model presented in the following is a generalization of the hadron-exchange model for the $N\overline{N} \to \pi\pi$ transition amplitudes of Ref. [17], where it was applied to the analysis of correlated two-pion exchange in the πN interaction. A main feature of the model presented here is the completely consistent treatment of its two components, namely the $B\overline{B'} \to \pi\pi, K\overline{K}$ Born amplitudes and the $\pi\pi - K\overline{K}$ correlations. Both components are derived in field theory from an ansatz for the hadronic Lagrangians.

The amplitudes for the processes $B\overline{B'} \to \pi\pi, K\overline{K}$ are obtained from a scattering equation which can be written in operator form as

$$\mathbf{T} = \mathbf{V} + \mathbf{TGV} \quad , \tag{63}$$

where the scattering amplitude $\mathbf{T}$, the Born amplitude $\mathbf{V}$ and the Greens operator $\mathbf{G}$ are operators in channel space of $B\overline{B'}$, $\pi\pi$ and $K\overline{K}$.

For large t', contributions of the spectral functions of correlated $\pi\pi$ and $K\overline{K}$ exchange to the dispersion integrals 48–50 values are suppressed by the denominator $1/(t'-t)$ because in the physical s-channel $t \leq 0$. Since the unitarity cuts of the $B\overline{B'}$ states start far above the branch points of the $\pi\pi$ and $K\overline{K}$ cuts the contribution of the $B\overline{B'}$ Greens function $G_{B\overline{B'}}$ to the $B\overline{B'} \to \pi\pi, K\overline{K}$ scattering amplitudes in Eq. 63 and to the spectral functions can be neglected. For the same reason the coupling to other mesonic channels like the $\rho\rho$ channel can be renounced. Of course, by these approximations the range of validity of the microscopic model for the $B\overline{B'} \to \mu\overline{\mu}$ ($\mu\overline{\mu} = \pi\pi, K\overline{K}$) amplitudes is limited to low t' values. Therefore instead of integrating along the whole $\pi\pi$ unitarity cut up to infinity the upper bound of the dispersion integrals, t'_{max}, is set to a value

somewhere below the $\rho\rho$ threshold at $t' \approx 120 m_\pi^2$ so that convergence of the integrals is achieved.

Taking into account only $\pi\pi$ and $K\overline{K}$ intermediate states, i.e. neglecting $G_{B\overline{B'}}$, we obtain for the two components of Eq. 63 which give the $B\overline{B'} \rightarrow \mu\overline{\mu}$ amplitudes:

$$\begin{pmatrix} T_{B\overline{B'}\rightarrow\pi\pi} \\ T_{B\overline{B'}\rightarrow K\overline{K}} \end{pmatrix} = \begin{pmatrix} V_{B\overline{B'}\rightarrow\pi\pi} \\ V_{B\overline{B'}\rightarrow K\overline{K}} \end{pmatrix} + \begin{pmatrix} T_{\pi\pi\rightarrow\pi\pi} & T_{K\overline{K}\rightarrow\pi\pi} \\ T_{\pi\pi\rightarrow K\overline{K}} & T_{K\overline{K}\rightarrow K\overline{K}} \end{pmatrix} \begin{pmatrix} G_{\pi\pi}\, V_{B\overline{B'}\rightarrow\pi\pi} \\ G_{K\overline{K}}\, V_{B\overline{B'}\rightarrow K\overline{K}} \end{pmatrix} . \tag{64}$$

In Fig. 5 this scattering equation for the $B\overline{B'} \rightarrow \pi\pi, K\overline{K}$ amplitudes is represented symbolically.

The correlations $T_{\pi\pi/K\overline{K}\rightarrow\pi\pi/K\overline{K}}$ are generated with a realistic model [17] of the $\pi\pi - K\overline{K}$ interaction. This meson-exchange model is a modified version of the so-called Jülich $\pi\pi$ model [13]. The Born amplitudes of this model for the elastic $\pi\pi$ and $K\overline{K}$ channels as well as for the transition $\pi\pi \rightarrow K\overline{K}$ are shown in Fig. 6. Besides the t-channel (and in case of $\pi\pi \rightarrow \pi\pi$ also u-channel) exchanges of the vector mesons ρ, ω, ϕ, K^* the s-channel exchanges (pole graphs) of the ρ, the scalar-isoscalar ϵ and the isoscalar tensor meson f_2 are taken into account. The potentials derived from Fig. 6 are iterated in a coupled-channel calculation according to the prescription of Blankenbecler and Sugar [31]. The free parameters of the $\pi\pi - K\overline{K}$ model of Ref. [17] were adjusted to the empirical $\pi\pi$ phase shifts and inelasticities. A good agreement with the empirical data was achieved(cf. Fig. 8).

The scattering equation 64 is likewise solved using the ansatz [31] of Blankenbecler and Sugar (BbS) to reduce the 4-dimensional Bethe-Salpeter equation to a 3-dimensional equation which simplifies the calculation while retaining unitarity.

The total conserved 4-momentum P and the relative 4-momentum k' of the intermediate $\mu'\overline{\mu'} = \pi\pi, K\overline{K}$ state are expressed in terms of the particle 4-momenta $k'(1)$ and $k'(2)$ by

$$P = k'(1) + k'(2) \quad , \qquad k' = (k'(1) - k'(2))/2 \quad . \tag{65}$$

Corresponding relations hold for the relative 4-momenta q of the initial $B\overline{B'}$ and k of the final $\mu\overline{\mu} = \pi\pi, K\overline{K}$ state. In the center-of-mass system we have $P = (\sqrt{t'}, \vec{0})$ with $\sqrt{t'} = E_B(q) + E_{B'}(q)$, $\vec{k}' = \vec{k}'(1) = -\vec{k}'(2)$ and, if the particles in the initial and final state are on their mass-shell,

$$q_0 = (E_B(q) - E_{B'}(q))/2 \quad , \qquad k_0 = k'_0 = 0 \tag{66}$$

with $E_B(q) := \sqrt{\vec{q}^{\,2} + M_B^2}$.

Now, according to the prescription of Blankenbecler and Sugar, the relativistic two-particle Greens function, $G_{\mu'\overline{\mu'}}$, is replaced by a 3-dimensional Greens function $g_{\mu'\overline{\mu'}} \propto \delta(k'_0)$, which respects unitarity. Due to the δ function $\delta(k'_0)$, which sets the two intermediate particles (being of equal mass) equally off-mass-shell ($k'_0(1) = k'_0(2)$), the integration over k'_0 can be carried out immediately and the so-called Blankenbecler-Sugar equation for the helicity amplitudes is obtained:

$$\begin{aligned} &\langle\, \mu\bar{\mu}, \vec{k} \,|\, T(t) \,|\, B\overline{B'}, \vec{q}, \lambda_1\lambda_2 \,\rangle = \langle\, \mu\bar{\mu}, \vec{k} \,|\, V(t) \,|\, B\overline{B'}, \vec{q}, \lambda_1\lambda_2 \,\rangle \\ &\quad + \frac{1}{(2\pi)^3} \sum_{\mu'\bar{\mu}'=\pi\pi, K\bar{K}} N_{\mu'\bar{\mu}'} \int d^3k' \frac{T_{\mu'\bar{\mu}'\rightarrow\mu\bar{\mu}}(\vec{k}, \vec{k}'; t) \langle\, \mu'\bar{\mu}', \vec{k}' \,|\, V(t) \,|\, B\overline{B'}, \vec{q}, \lambda_1\lambda_2 \,\rangle}{\omega_{k'}(t - 4\omega_{k'}^2 + i\epsilon)} \quad , \end{aligned} \tag{67}$$

where $N_{\mu'\bar{\mu}'}$ is defined as in Eq. 23 and $T_{\mu'\bar{\mu}'\to\mu\bar{\mu}} \equiv \mathcal{M}_{\mu'\bar{\mu}'\to\mu\bar{\mu}}$.

Because of the rotational invariance of the underlying interactions the BbS equation can be decomposed into partial waves [24]:

$$\begin{aligned}\langle\, \mu\bar{\mu}\,|\, T^J(k,q;t)\,|\, B\overline{B'}, \lambda_1\lambda_2\,\rangle &= \langle\, \mu\bar{\mu}\,|\, V^J(k,q;t)\,|\, B\overline{B'}, \lambda_1\lambda_2\,\rangle \\ &+ \frac{1}{(2\pi)^3}\sum_{\mu'\bar{\mu}'} N_{\mu'\bar{\mu}'}\int k'^2 dk' \frac{T^J_{\mu'\bar{\mu}'\to\mu\bar{\mu}}(k,k';t)\langle\, \mu'\bar{\mu}'\,|\, V^J(k',q,t)\,|\, B\overline{B'}, \lambda_1\lambda_2\,\rangle}{\omega_{k'}(t-4\omega^2_{k'}+i\epsilon)}.\end{aligned} \tag{68}$$

Here, q, k, k' denote the modulus of the corresponding 3-momenta.

The extrapolation of the model for the $N\overline{N} \to \pi\pi$ amplitudes to the other particle channels is made under the assumption that the hadronic interactions are, except for the particle masses, $SU(3)_{flavor}$ symmetric. That means that the coupling constants at the various hadronic vertices are related to each other by $SU(3)$ relations. In this way it comes out that all free parameters of the model for the $B\overline{B'} \to \mu\overline{\mu}$ amplitudes can be fixed by adjusting the $N\overline{N} \to \pi\pi$ amplitudes to the quasiempirical data [18, 19] (see Sect. 3.1).

2.4.1 $B\overline{B'} \to \pi\pi, K\overline{K}$ Born amplitudes

In our model, the $B\overline{B'} \to \mu\overline{\mu}$ transition potentials are build up from a ρ-pole diagram and all diagrams in which a baryon out of the $J^P = 1/2^+$ octet or the $J^P = 3/2^+$ decuplet is exchanged. Of course only those diagrams are considered which respect the conservation of isospin and strangeness. As an example Fig. 7 shows the Born amplitudes for the transition $\Sigma\overline{\Sigma} \to \pi\pi, K\overline{K}$. The particles occuring in this model are listed in Tab. 2 together with their masses and their basic quantum numbers.

In order to start from a maximum $SU(3)$ symmetry our model differs slightly from the one presented in Ref. [17] for the $N\overline{N} \to \pi\pi$ amplitudes. Here, we include the exchange of the $Y^* \equiv \Sigma(1385)$ in the $N\overline{N} \to K\overline{K}$ transition potential, which was neglected in Ref. [17]. In addition, the form factors at the hadronic vertices are chosen identical within a given $SU(3)$ multiplet.

Starting point for the derivation of the various Born amplitudes are the following interaction Lagrangians, which are characterized by the J^P quantum numbers of the hadrons involved. For simplicity, we suppress here the isospin or $SU(3)$ dependence of the Lagrangians.

$$\begin{aligned}
\tfrac{1}{2}^+ \otimes \tfrac{1}{2}^+ \otimes 0^- :&\quad \mathcal{L}_{B'Bp}(x) = \tfrac{f_{B'Bp}}{m_{\pi^+}}\overline{\psi}_{B'}(x)\gamma_5\gamma^\mu\psi_B(x)\partial_\mu\phi_p(x) \quad (+\text{ h.c., if } B' \neq B)\,,\\
\tfrac{3}{2}^+ \otimes \tfrac{1}{2}^+ \otimes 0^- :&\quad \mathcal{L}_{DBp}(x) = \tfrac{f_{DBp}}{m_{\pi^+}}\overline{\psi}_B(x)(g_{\mu\nu} + x_\Delta\gamma_\mu\gamma_\nu)\psi^\mu_D(x)\partial^\nu\phi_p(x) \quad +\text{ h.c.}\,,\\
\tfrac{1}{2}^+ \otimes \tfrac{1}{2}^+ \otimes 1^- :&\quad \mathcal{L}_{B'B\rho}(x) = g_{B'B\rho}\overline{\psi}_{B'}(x)\gamma_\mu\psi_B(x)\phi^\mu_\rho(x)\\
&\qquad\qquad + \tfrac{f_{B'B\rho}}{4M_N}\overline{\psi}_{B'}(x)\sigma_{\mu\nu}\psi_B(x)(\partial^\mu\phi^\nu_\rho(x) - \partial^\nu\phi^\mu_\rho(x))\\
&\qquad\qquad (+\text{h.c., if } B' \neq B)\,,\\
0^- \otimes 0^- \otimes 1^- :&\quad \mathcal{L}_{p'p\rho}(x) = g_{p'p\rho}\phi_{p'}(x)\partial_\mu\phi_p\phi^\mu_\rho(x) \quad (+\text{h.c., if } p' \neq p)\,.
\end{aligned} \tag{69}$$

For the conventions used for the field operators and the Dirac γ-matrices, see Appendix A. The Lagrangian $\mathcal{L}_{DBp}$ includes an off-shell part, which is proportional to x_Δ. As the parameter A, which occurs in the free Lagrangian [32] and then in the non-pole part of the propagator of a spin-3/2 particle (cf. Eq. 101), the parameter x_Δ characterizing the strength of the off-shell part of the DBp coupling is not determined from first principles. However, it is known [33] that fieldtheoretic amplitudes derived with this most general ansatz depend only on a certain combination of A and x_Δ, namely

$$\frac{1+4x_\Delta}{2A+1} =: 1+4Z \quad . \tag{70}$$

It follows that different pairs of (x_Δ, A) values, which give the same value of Z, describe the same interaction theory. Therefore, without restricting the general validity of our results, we can set $A=-1$ (i.e. omitting the non-pole part of the spin-3/2 propagator) and select the interaction theory (characterized by Z) through x_Δ, which is finally adjusted to the quasiempirical data for $N\overline{N} \to \pi\pi$.

In order to account for the extended structure of hadrons the vertex functions resulting from the Lagrangians 69 are modified by phenomenological form factors. For the baryon-exchange processes these form factors are parametrized in the usual multipole form

$$F_X(p^2) = \left(\frac{n_X\Lambda_X^2 - M_X^2}{n_X\Lambda_X^2 - p^2}\right)^{n_X} \quad , \tag{71}$$

where p denotes the 4-momentum and M_X the mass of the exchanged baryon X. The two parameters, the so-called cutoff mass Λ_X and the power n_X, are chosen uniquely for all $BB'p$ (Λ_8, n_8) and for all BDp (Λ_{10}, n_{10}) vertices in order to keep the number of parameters of our model as low as possible. The dependence of the form factor 71 on the power n_X is quite weak [17]. We choose $n_8=1$ and $n_{10}=2$. Finally, Λ_8 and Λ_{10} are adjusted to the quasiempirical data for the $N\overline{N} \to \pi\pi$ amplitudes in the pseudophysical region.

For the ρ-pole diagram we parametrize the form factor at the $\mu\mu\rho$ vertex in the same way as is done in the model of the $\pi\pi$-$K\overline{K}$ interaction in Ref. [17]:

$$F_{\mu\mu\rho}(\vec{k}^{\,2}) \quad = \quad \left(\frac{n_{\mu\mu\rho}\Lambda_{\mu\mu\rho}^2 + (m_\rho^{(0)})^2}{n_{\mu\mu\rho}\Lambda_{\mu\mu\rho}^2 + 4\omega_\mu^2(\vec{k}^{\,2})}\right)^{n_{\mu\mu\rho}} \tag{72}$$

where $\vec{k}$ is the relative 3-momentum of the two pseudoscalar mesons and $k_{\mu\bar{\mu}}^2(t) = t/4 - m_\mu^2$ is the squared on-shell momentum of the $\mu\overline{\mu}$ state.

For the dispersiontheoretic calculation of correlated $\pi\pi$ and $K\overline{K}$ exchange the $B\overline{B'} \to \mu\overline{\mu}$ Born amplitudes are evaluated only for $B\overline{B'}$ states being on their mass-shell. Therefore, there is no need for a form factor at the $BB'\rho$ vertex as to assure convergence of the scattering equation 68. Therefore, we disregard this form factor, again in order to keep the parameters of the model as low as possible.

Now, taking into account that the $B\overline{B'}$ state is on mass-shell and that due to the Blankenbecler-Sugar condition (Eq. 66) the energy component of the relative momentum of the $\mu\overline{\mu}$ state always vanishes ($k_0 = 0$) the 4-momenta at the external legs of the $B\overline{B'} \to \mu\overline{\mu}$ Born diagrams read in the center-of-mass system:

$$q_B = \begin{pmatrix} E_B(q) \\ \vec{q} \end{pmatrix}, \; q_{\bar{B}'} = \begin{pmatrix} E_{B'}(q) \\ -\vec{q} \end{pmatrix}, \quad k(\mu) = \begin{pmatrix} \sqrt{t'} \\ \vec{k} \end{pmatrix}, \; k(\overline{\mu}) = \begin{pmatrix} \sqrt{t'} \\ -\vec{k} \end{pmatrix}. \tag{73}$$

According to the usual Feynman rules [34] (for the various propagators, see also Appendix A) we obtain for the spin-momentum parts of the $B\overline{B'} \to \mu\overline{\mu}$ Born amplitudes $\mathcal{V}_{B\overline{B'}\to\mu\overline{\mu}}(\vec{k},\vec{q};t)$:

- Exchange of a baryon X with $j^P = \frac{1}{2}^+$ and momentum $p = q - k$

$$\mathcal{V}^X_{B\overline{B'}\to\mu\overline{\mu}}(\vec{k},\vec{q};t') = \left(-\frac{f_{XB'\mu}}{m_{\pi^+}}\gamma_5\gamma_\lambda k^\lambda(\overline{\mu})\right)\frac{-(\not{p}+M_X)}{p^2 - M_X^2}\left(-\frac{f_{XB\mu}}{m_{\pi^+}}\gamma_5\gamma_\nu k^\nu(\mu)\right) \quad , \tag{74}$$

- Exchange of a baryon X with $J^P = \frac{3}{2}^+$ and momentum $p = q - k$

$$\mathcal{V}^X_{B\overline{B'}\to\mu\overline{\mu}}(\vec{k},\vec{q};t') = \left[-\frac{f_{XB'\mu}}{m_{\pi^+}}(g_{\lambda\nu} + x_\Delta\gamma_\nu\gamma_\lambda)k^\nu(\overline{\mu})\right] S_X^{\lambda\rho}(p, A=-1)$$
$$\left[-\frac{f_{XB\mu}}{m_{\pi^+}}(g_{\rho\sigma} + x_\Delta\gamma_\rho\gamma_\sigma)k^\sigma(\mu)\right] \quad , \tag{75}$$

- ρ-pole graph with bare mass $m_\rho^{(0)}$

$$\mathcal{V}^\rho_{B\overline{B'}\to\mu\overline{\mu}}(\vec{k},\vec{q};t') = \left(ig^{(0)}_{BB'\rho}\gamma_\lambda + \frac{f^{(0)}_{BB'\rho}}{2M_N}\sigma_{\lambda\nu}P^\nu\right)\frac{g^{\lambda\sigma} - P^\lambda P^\sigma/(m_\rho^{(0)})^2}{t' - (m_\rho^{(0)})^2}$$
$$\left[-ig^{(0)}_{\mu\mu\rho}(k(\mu) - k(\overline{\mu}))_\sigma\right] . \tag{76}$$

These expressions can be further simplified by introducing the momenta given in Eq. 73 and contracting the γ-matrices. Finally, the corresponding $B\overline{B'} \to \mu\overline{\mu}$ helicity amplitudes are obtained by applying $\mathcal{V}_{B\overline{B'}\to\mu\bar{\mu}}(\vec{k},\vec{q};t)$ to the Dirac helicity spinors of the baryons (cf. Eqs. 87 and 94 in the Appendix):

$$\langle\, \mu\bar{\mu}, \vec{k}\,|\, V(t)\,|\, B\overline{B'}, \vec{q}, \lambda_1\lambda_2\,\rangle = \overline{v}_{B'}(-\vec{q},\lambda_2)\mathcal{V}_{B\overline{B'}\to\mu\bar{\mu}}(\vec{k},\vec{q};t)u_B(\vec{q},\lambda_1) \quad . \tag{77}$$

The final results for the helicity amplitudes are summarized in Appendix B.

In our model the coupling constants at the various hadronic vertices are related to each other by $SU(3)$ arguments. The $SU(3)$ relations together with the isospin factors of the various Born amplitudes are given in Appendix C.

As discussed in the previous chapters, for the dispersiontheoretic calculation of correlated $\pi\pi$ and $K\overline{K}$ exchange the $B\overline{B'} \to \pi\pi, K\overline{K}$ have to be known in the pseudophysical region, i.e. for energies t' below the $B\overline{B'}$ threshold ($4m_\pi^2 \le t' < (M_B + M_{B'})^2$). Therefore, after having derived the analytic expressions for these Born amplitudes in the physical region ($\sqrt{t'} > M_B + M_{B'}$) they have to be continued analytically as functions of t' (and s) into the pseudophysical region. For this all energy-dependent quantities occuring in the expressions for the Born amplitudes (cf. Appendix B) have to be expressed as functions of t' and s.

If we adopt the approximation introduced in Sect. 2.2, namely that the masses of the baryon and of the antibaryon are equal ($M_B = M_{B'}$), the square of the relative 4-momentum and the one-particle energies of the $B\overline{B'}$ state are given in the center-of-mass system for physical values of t by

$$\begin{aligned} q^2(t) &= t/4 - M_B^2 \quad , \\ E_B = E_{B'} &= \sqrt{t}/2 \quad . \end{aligned} \tag{78}$$

The analytic continuation of these relations to the pseudophysical region is obvious. Note that if we would have allowed M_B and $M_{B'}$ to be different the corresponding relations would look more involved:

$$
\begin{aligned}
q^2(t) &= \frac{\left[t-(M_B+M_{B'})^2\right]\left[t-(M_B-M_{B'})^2\right]}{4t} \quad , \\
E_B(t) &= \frac{(E_B+E_{B'})}{2} + \frac{(E_B^2-E_{B'}^2)}{2(E_B+E_{B'})} \\
&= \frac{t+M_B^2-M_{B'}^2}{2\sqrt{t}} \quad , \\
E_{B'}(t) &= \frac{t+M_{B'}^2-M_B^2}{2\sqrt{t}} \quad .
\end{aligned}
\tag{79}
$$

Baryon-exchange diagrams in which the mass M_X of the exchanged baryon is sufficiently smaller than the mass $M_B = M_{B'}$ of the external baryons (e.g. N-exchange in $\Lambda\overline{\Lambda} \to K\overline{K}$ or Λ-exchange in $\Sigma\overline{\Sigma} \to \pi\pi$ do not satisfy a Mandelstam representation as was already pointed out in Ref. [35] for the latter example. The nonvalidity of the Mandelstam representation becomes obvious when extrapolating the corresponding Born amplitude in Eq. 74 to the pseudophysical region. For given $t < 4(M_B^2 - M_X^2)$, the propagator of baryon X,

$$
\left[p^2 - M_X^2\right]^{-1} = -\left[t/4 - M_B^2 + M_X^2 + \vec{k}^2 - 2\vec{q}\cdot\vec{k}\right]^{-1} \quad , \tag{80}
$$

then acquires a singularity at $cos\theta = 0$ ($\vec{q}\cdot\vec{k}$ imaginary) and the following off-shell momentum of the $\mu\overline{\mu}$ state

$$
\vec{k}^2 = M_B^2 - M_X^2 - t/4 > 0 \quad . \tag{81}
$$

Since this problem would hinder an evaluation of correlated $\pi\pi$ and $K\overline{K}$ exchange and we do not see at present any proper solution, we eliminate the problem by the following approximation: In all baryon-exchange diagrams in which the mass of the exchanged baryon, M_X, is smaller than the mass of the external baryons, M_B, M_X is increased by hand to M_B. Again, since the uncorrelated contributions of two-pion and two-kaon exchange (e.g. iterative two-pion exchange in the ΣN channel with a ΛN intermediate state) are evaluated explicitly in the s-channel, these contributions are not affected by our approximation and thus have the correct energy dependence. Approximations are only made in the correlated part which have a much weaker energy dependence than the uncorrelated contributions.

3 Results and Discussion

3.1 Determination of free parameters

During the construction of the microscopic model for the $B\overline{B'} \to \mu\overline{\mu}$ amplitudes it proved to be essential to restrict the number of free parameters as much as possible, which can then be fixed by adjusting the model predictions to the quasiempirical $N\overline{N} \to \pi\pi$ amplitudes. Only in this way one can hope to obtain a reasonable description of the other baryon-antibaryon channels, for which no empirical data exist.

As shortly outlined in Sect. 2.4 the $\pi\pi - K\overline{K}$ interaction model has been developed independently before, with all parameters adjusted to fit the existing $\pi\pi$ scattering phase

shifts. Therefore coupling constants and form factors at the $\pi\pi\rho^{(0)}$ and $KK\rho^{(0)}$ vertices occurring in the $\rho^{(0)}$ pole terms are already determined. Assuming that the bare ρ meson couples universally to the isospin current we can also fix all vector couplings $g^{(0)}_{BB'\rho}$ to the baryonic vertices. Corresponding tensor couplings $f^{(0)}_{BB'\rho}$ will be related by $SU(3)$ symmetry, with two parameters remaining, namely the coupling constant $f^{(0)}_{NN\rho}$ and the $F/(F+D)$ ratio α_v^m.

The coupling of the pseudoscalar mesons π and K to the octet baryons is in the framework of $SU(3)$ symmetry likewise determined by two parameters, the $F/(F+D)$ ratio α_p and the coupling constant $g_{NN\pi}$. (For the latter we will take throughout $g^2_{NN\pi}/4\pi = 14.3$). There is an additional freedom since $SU(3)$ symmetry can be either assumed for the pseudoscalar coupling constants $g_{BB'p}$ or the pseudovector ones $f_{BB'p}$, which are related by

$$g_{B'Bp} = f_{B'Bp}\,\frac{M_B + M_{B'}}{m_{\pi^+}} \quad . \tag{82}$$

These two possibilities are not equivalent since, because of $SU(3)$ breaking of baryon masses, both sets of coupling constants cannot be $SU(3)$ symmetric at the same time. In this work we will assume $SU(3)$ symmetry for the pseudoscalar couplings, but we will also check the influence of the alternative possibility.

Finally, under the assumption of $SU(3)$, the couplings f_{BDp} of π and K to the transition current between baryon octet and decuplet are determined by only one parameter, $f_{N\Delta\pi}$. We will use $f^2_{N\Delta\pi}/4\pi = 0.36$ in the following.

In addition we have form-factor parameters at the hadronic vertices. In order to keep their number small we assume that the cutoff masses $\Lambda_{BX\mu}$ are independent of the exchanged baryon X within one $SU(3)$ multiplet. Consequently we have two additional parameters: Λ_8 if X is a member of the baryon octet and Λ_{10} if it is in a decuplet. The power $n_8 = 1$ and $n_{10} = 2$ in the form-factor *ansatz* are sufficient to ensure convergence of the scattering equation. Finally, x_Δ (Eq. 69) characterizing the off-shell part of the $(3/2)^+ \otimes (1/2)^+ \otimes 0^-$ coupling is treated as a free parameter in our $B\overline{B'} \to \mu\overline{\mu}$ model.

In order to reduce the number of parameters further we even assume $SU(6)$ symmetry, which fixes α_p and α_v^m to be 0.4. Thus we are left with four free parameters $f^{(0)}_{NN\rho}$, x_Δ, Λ_8 and Λ_{10}, which have been fixed by adjusting our theoretical predictions for the $N\overline{N} \to \pi\pi$ amplitudes to the quasiempirical results of Höhler and Pietarinen [18, 19] given in the form of Frazer-Fulco amplitudes $f^J_\pm(t)$, which, up to kinematic factors, correspond to the partial wave decomposed helicity amplitudes of Sect. 2.2, i.e.

$$\begin{aligned} f^J_+(t) &= -\frac{1}{16\pi^2}\frac{qM_N}{(qk)^J}\langle\, \pi\pi \,|\, T^J(t) \,|\, N\overline{N}, ++ \,\rangle \times F^J \quad , \\ f^J_-(t) &= -\frac{1}{8\pi^2}\frac{qM_N}{(qk)^J\sqrt{t}}\langle\, \pi\pi \,|\, T^J(t) \,|\, N\overline{N}, +- \,\rangle \times F^J \quad , \end{aligned} \tag{83}$$

where k and q are the on-shell momenta of pions and nucleons. The factors $F^J = -1/\sqrt{6}\,(-1/2)$ for $J = 0\,(1)$ are due to the transition from isospin amplitudes used in this work to the Frazer-Fulco amplitudes, which are defined in isospin space as coefficients of the independent isospin operators $\delta_{\alpha\beta}$ and $\frac{1}{2}[\tau_\alpha, \tau_\beta]$. Since $\langle\, \pi\pi \,|\, T^{J=0}(t) \,|\, N\overline{N}, +- \,\rangle$ vanishes identically we have only one amplitude, f^0_+, in the σ channel whereas in the ρ channel we have both f^1_+ and f^1_-.

Fig. 9 shows the predictions of our microscopic model for f_+^0, f_+^1 and f_-^1, in comparison to the quasiempirical results of Ref. [18] in the pseudophysical region $t \geq 4m_\pi^2$; Table 4 contains the chosen parameter values. (Note that the present model differs somewhat from our former model [17], e.g. by the inclusion of Y^* exchange; therefore the values differ slightly from those given in Ref. [17]). With only four parameters we obtain a very satisfactory reproduction of the quasiempirical data, especially in the ρ channel. Some discrepancies occur in the σ channel, which however have only small influence on final results for the correlated $\pi\pi$ exchange in the s-channel reactions (NN, πN), as also discussed below. Furthermore it should be kept in mind that in this channel the quasiempirical information is plagued with considerable uncertainties.

3.2 The $B\overline{B'} \rightarrow \mu\overline{\mu}$ transition potentials

Having fixed all parameters in our microscopic model for the $B\overline{B'} \rightarrow \mu\overline{\mu}$ amplitudes we will now first look at the transition potentials (Born amplitudes) in the various baryon-antibaryon channels. Figs. 10 and 11$B\overline{B'} \rightarrow \pi\pi$, $K\overline{K}$ amplitudes show the contributions of the various baryon-exchange processes to the $N\overline{N} \rightarrow \pi\pi, K\overline{K}$ and $\Sigma\overline{\Sigma} \rightarrow \pi\pi, K\overline{K}$ Born amplitudes above the $\mu\overline{\mu}$ thresholds, i.e. for $t \geq 4m_\mu^2$. Contributions of the $\rho^{(0)}$ pole terms are not shown since they possess a singularity at the bare rho mass $m_\rho^{(0)}$. This pole will be regularized only after iteration and coupling to the full $\pi\pi$ amplitude and then leads to the resonance structure at the physical rho mass m_ρ.

The notation of the (Born) amplitudes follows that of the Frazer-Fulco amplitudes, i.e.

$$\begin{aligned} V_+^J(t) &= \langle\, \mu\overline{\mu} \,|\, V^J(t) \,|\, B\overline{B'}, ++ \,\rangle \quad , \\ V_-^J(t) &= \langle\, \mu\overline{\mu} \,|\, V^J(t) \,|\, B\overline{B'}, +- \,\rangle \quad . \end{aligned} \tag{84}$$

The partial wave decomposed Born amplitudes are either even or odd functions of the baryonic relative momenta (cf. Appendix B). Since the latter are imaginary in the pseudophysical region the Born amplitudes being odd functions (e.g. V_+^0) become likewise imaginary.

Apart from coupling constants and isospin factors the strengths of the various contributions are strongly determined by mass ratios. Exchange baryons with the same mass M_X as the outer baryon-antibaryon pairs (according to our approximation introduced in Sect. 2.4.1 this case includes also those exchange baryons whose physical mass is lower than that of the outer baryons) produce, in the $\pi\pi$ amplitudes V_+^0 and V_+^1, a typical structure at the $\pi\pi$ threshold. The strong rise of the amplitudes, which acquire a finite value at $t = 4m_\pi^2$, is a direct signal of the so-called left-hand cut, which is generated by the singularity of the corresponding u-channel pole graph. This cut starts just below the $\pi\pi$ threshold and extends from $t_0 = 4m_\pi^2(1 - m_\pi^2/M_X)$ along the real axis to $-\infty$.

Obviously, in the σ channel, the various pieces interfere constructively, whereas in the ρ channel also destructive interferences occur. The contributions generated by a spin-3/2 exchange baryon have opposite sign in V_+^1 and V_-^1 whereas both amplitudes are almost equal when a spin-1/2 baryon is exchanged.

The $N\overline{N} \rightarrow \pi\pi$ Born amplitudes in Fig. 10 built up by nucleon and Δ exchange are noticeably larger than the $N\overline{N} \rightarrow K\overline{K}$ Born amplitudes, which are suppressed because of the high mass of the exchange baryons Λ, Σ, Y^*. Due to different mass ratios this is no longer true in the hyperon-antihyperon channels. For instance, in the $\Sigma\overline{\Sigma}$ channel

shown in Fig. 11 the coupling via Δ exchange to $K\overline{K}$ even dominates, due to the large coupling constant ($f^2_{\Sigma\Delta K} = f^2_{N\Delta\pi}$) and the small mass difference between Σ and Δ ($M_\Delta - M_\Sigma \approx 39\ MeV$). Therefore already from these results it is to be expected that correlated $K\overline{K}$ exchange processes play a minor role in the NN system but are important for the interactions involving hyperons.

3.3 The $B\overline{B'} \to \mu\overline{\mu}$ transition amplitudes

The $B\overline{B'} \to \mu\overline{\mu}$ helicity amplitudes, $T^J_\pm(t)$, which are obtained from the solution of the BbS scattering equation 68, consist of Born terms and pieces containing meson-meson correlations.The latter part fixes the phase of the $B\overline{B'} \to \mu\overline{\mu}$ amplitudes and generates the discontinuities of $T^J_\pm(t)$ along the unitarity cut. It contains $\pi\pi$ and $K\overline{K}$ correlations in the form of $\pi\pi$, $K\overline{K}$ amplitudes. Corresponding predictions from our field-theoretic model, for the scalar-isoscalar on-shell amplitude $T^{J=0,I=0}(t)$ are shown in Fig. 12. By comparison of the $\pi\pi$ amplitude with the $\pi\pi$ phase shifts (cf. Fig. 8) we can see immediately that the vanishing of the real part of the amplitude at $t = 37m^2_\pi$ and $t = 71m^2_\pi$ corresponds to the phase shifts acquiring the value of 90 resp. 270 degrees, respectively. The phase value of 180 degrees (and the corresponding vanishing of the amplitude) occurs just below the $K\overline{K}$ threshold ($t = 50.48m^2_\pi$). Remarkably, there is a steep rise of the imaginary part of the $\pi\pi$ amplitude at low energies, which has to vanish at the $\pi\pi$ threshold for unitarity reasons. We mention that, in the NN system, this part provides the dominant contribution to correlated $\pi\pi$ exchange. It should be noted that both the real and imaginary part of the $K\overline{K} \to K\overline{K}$ amplitude (which in Fig. 12 are shown only in the physical region above the $K\overline{K}$ threshold) acquire a non-vanishing value at the $K\overline{K}$ threshold due to the open $\pi\pi$ channel.

Figs. 13 and 14 show the resulting $B\overline{B'} \to \pi\pi$ amplitudes for $B\overline{B'} = N\overline{N}, \Sigma\overline{\Sigma}$. Besides the predictions of the full model (solid line) the figures also show the effect which is obtained when neglecting the $B\overline{B'} \to K\overline{K}$ Born amplitudes in the σ channel (dashed line) and restricting oneself to the ρ-pole amplitudes in the ρ channel (dash-dotted line). The $B\overline{B'} \to K\overline{K}$ amplitudes (dotted lines) are only shown for $J = 0$ since for $J = 1$ as exemplified for the $\Sigma\overline{\Sigma}$ channel they provide almost negligible contributions, mainly because of the weak $K\overline{K}$ interaction in that partial wave. Note that in the σ channel, due to the imaginary Born amplitudes, the role of the real and imaginary part of T^0_+ are in a certain sense interchanged: $Im[T^0_+]$ contains the Born amplitudes whereas the discontinuity along the unitarity cut is contained in $Re[T^0_+]$.

Obviously, in all channels, the structure of the $\pi\pi$ amplitude of Fig. 12 can be recognized in those $B\overline{B'} \to \pi\pi$ amplitudes T^0_+, which have been evaluated without the $B\overline{B'} \to K\overline{K}$ Born amplitudes (dashed curves). The steep rise of $Im[T^0_+]$ at the $\pi\pi$ threshold originates from the analogous behavior of the Born amplitudes (cf. Figs. 10 and 11). As follows from the unitarity relation for the $B\overline{B'} \to \pi\pi$ amplitudes below the $K\overline{K}$ threshold ($t_{thr} \approx 50.48m^2_\pi$) the discontinuity of the $B\overline{B'} \to \pi\pi$ amplitudes (contained in $Re[T^0_+]$) must vanish at $t \approx 50.43m^2_\pi$ since there $T^{J=0,I=0} = 0$. Furthermore, those amplitudes T^0_+, which have been evaluated without the $K\overline{K}$ Born amplitudes, vanish at an additional, though for each channel different position in the energy region below the $K\overline{K}$ threshold. Interestingly this vanishing of the amplitudes is lost in the hyperon-antihyperon channels when the $K\overline{K}$ Born amplitudes are included; in case of the $N\overline{N} \to \pi\pi$ channel on the other hand the resulting full amplitude still vanishes, at a slightly

different position ($t \approx 50.2m_\pi^2$). Note that the quasiempirical $N\overline{N} \to \pi\pi$ amplitude f_+^0 in Fig. 9 has a comparable structure, however the amplitude vanishes already at a somewhat smaller value of $t \approx 43m_\pi^2$. Fig. 15 also clearly shows the cusp structure of the amplitudes at the $K\overline{K}$ threshold, which is however much stronger in the hyperon-antihyperon channels.

The $B\overline{B'} \to K\overline{K}$ Born amplitudes do not play any role in the $N\overline{N} \to \pi\pi$ channel, as noted already in Refs. [14, 17]. In the hyperon-antihyperon channels on the other hand, we have already at small t-values important contributions to the $B\overline{B'} \to \pi\pi$ amplitudes T_+^0, which lead to an enhancement of the amplitudes below the $K\overline{K}$ threshold and to a noticeably different energy dependence.

The amplitudes $T_\pm^1$ in the ρ channel are dominated by the resonance structure in the region of the physical ρ mass at $t = 30.4m_\pi^2$. We stress that by multiplication with the $\pi\pi$ correlations in the scattering equation also the baryon-exchange Born terms lead to resonant contributions. However, in all channels, the discontinuity of the amplitudes mainly arises (to at least 60%) from both ρ pole terms and those contributions generated by them in the scattering equation (dash-dotted curves in Figs. 13 and 14).

As shown in Fig. 14 for the $\Sigma\overline{\Sigma} \to K\overline{K}$ transition, the $B\overline{B'} \to K\overline{K}$ on-shell amplitudes (dotted curves) are completely unimportant in the ρ channel. On the other hand, in the σ channel, the $B\overline{B'} \to K\overline{K}$ amplitudes can acquire large values especially near the $K\overline{K}$ threshold, due to the corresponding behavior of the $K\overline{K}$ amplitude $T^{J=0,I=0}$ (cf. Fig. 12), which lead to non-negligible contributions to the spectral functions.

3.4 The spectral functions

Based on the $B\overline{B'} \to \mu\overline{\mu}$ amplitudes in the pseudophysical region determined in the last section we can now in principle evaluate the spectral functions $\rho_i^{\sigma,\rho}$ (defined in Eq. 51) of correlated $\pi\pi$ and $K\overline{K}$ exchange for all baryon-baryon channels containing the octet baryons N, Λ, Σ and Ξ. In this work however we restrict ourselves to those channels in which experimental information is available at present or in the near future. Besides the baryon-baryon channels with strangeness $S = 0$ and $S = -1$, for which numerous (NN) and scarce ($N\Lambda$, $N\Sigma$) scattering data exist, we will also consider the $S = -2$ channels $\Lambda\Lambda$, $\Sigma\Sigma$ and $N\Xi$, which are relevant both for the description of $\Lambda\Lambda$- resp. Ξ-hypernuclei and for the study of the H-dibaryon [11] predicted in the ($S = -2$, $J = I = 0$) channel.

Fig. 16a (Fig. 16b) shows the spectral function $\rho_S^\sigma(BB')$ in the σ channel predicted by the full microscopic model for the $S = 0, -1$ BB' channels $NN, N\Lambda, N\Sigma$ ($S = -2$ channels $\Lambda\Lambda, \Sigma\Sigma, N\Xi$). Due to isospin conservation the $\pi\pi$ resp. $K\overline{K}$ exchange contributes to the $N\Lambda$ and $\Lambda\Lambda$ interaction only in the σ channel and to the $N\Lambda - N\Sigma$ transition only in the ρ channel. Up to near the $K\overline{K}$ threshold at $t = 50.48m_\pi^2$ ρ_S^σ is negative in all channels. Therefore, as expected, correlated $\pi\pi$ and $K\overline{K}$ exchange provides attractive contributions in all channels. Especially at small t-values, which determine the long-range part of the correlated exchanges and therefore yield for low-energy processes in the s-channel the main contributions to the dispersion integral, the spectral function $\rho_S^\sigma(NN)$ is by about a factor of 2 larger than the results for $\rho_S^\sigma(N\Lambda)$ and $\rho_S^\sigma(N\Sigma)$, which have about the same size. Due to the sizable contributions of the $K\overline{K}$ Born amplitudes to the hyperon-antihyperon amplitudes found already in the last section, $\rho_S^\sigma(N\Lambda)$ as well as $\rho_S^\sigma(N\Sigma)$ show a noticeably different t-dependence than $\rho_S^\sigma(NN)$. Below the $K\overline{K}$ threshold the spectral functions are broadened to larger t-values, therefore the overall

range of this correlated exchange is reduced. This is even more true for the $S = -2$ channels $\Lambda\Lambda$ and $\Sigma\Sigma$ in Fig. 16b since there the hyperon-antihyperon amplitudes enter quadratically. The effect of the $B\overline{B'} \to K\overline{K}$ Born amplitudes is once more shown in Fig. 17 for the NN and $N\Sigma$ channel. Whereas they have small influence on $\rho^{\sigma}_{S}(NN)$ they provide important contributions to $\rho^{\sigma}_{S}(N\Sigma)$ already near the $\pi\pi$ threshold.

Fig. 18 shows the corresponding spectral functions in the ρ channel (ρ^{ρ}_{S}, ρ^{ρ}_{V}, ρ^{ρ}_{T} and ρ^{ρ}_{6}) for the various particle channels, which are non-vanishing. (Note that ρ^{ρ}_{P} depends linearly on the four functions shown, cf. Eq. 37, and is therefore not shown). ρ^{ρ}_{6} contributes only in BB' channels with different masses and therefore vanishes in the NN and $\Sigma\Sigma$ channel. According to Eq. 35 $\rho^{\rho}_{S}(s,t)$ depends on s through the factor $\cos\vartheta_t(s,t)$. In order to enable a comparison of the results for $\rho^{\rho}_{S}(s,t)$ in the various particle channels we have throughout set s on the threshold of the s-channel process in question, i.e. $s = (M_B + M_{B'})^2$. Since the ρ channel is dominated by the resonant pieces in the $\pi\pi$ channel and the $K\overline{K}$ channel does not play any role the $\rho^{\rho}_{i}(BB')$ possess almost the same t-dependence in the various particle channels.

For the NN channel, the spectral functions for correlated $\pi\pi$ and $K\overline{K}$ exchange can be derived either from the $N\overline{N} \to \pi\pi$ amplitudes of our microscopic model or, alternatively, from the quasiempirical results obtained in Refs. [18, 19]. As before we have then to subtract the uncorrelated pieces evaluated from the microscopic model for the $N\overline{N} \to \pi\pi$ Born amplitudes. Therefore the results derived from the quasiempirical amplitudes depend on parameters (e.g. $g_{NN\pi}$) of the microscopic model.

In Fig. 19 we show the NN spectral functions in the σ as well as in the ρ channel obtained from our microscopic model (solid line) and the quasiempirical amplitudes (dashed line). As expected already from the comparison of the amplitudes in Sect. 3.1 the results agree quite well in the ρ channel. On the other hand some discrepancies occur in the σ channel. At smaller t-values, which determine the long-range part of the correlated exchanges, the theoretical model yields somewhat more attraction than the quasiempirical results. Larger discrepancies occur at higher t-values, which are however of minor relevance for the correlated exchange in the NN interaction. Namely, above $t \approx 30m_\pi^2$ the correlated exchanges are of shorter range than ω exchange, which generates the strong repulsive inner part of the NN potential (as well as of the other baryon-baryon potentials). Therefore the short-ranged parts of the correlated $\pi\pi$ exchanges are completely masked by the repulsive ω exchange and have a small influence only on NN observables. Furthermore one has to realize that the quasiempirical results obtained by extrapolation of data from the physical region of the s- and u-channel into the pseudophysical region of the t-channel have considerable uncertainties.

3.5 The potential of correlated $\pi\pi$ and $K\overline{K}$ exchange

Using the spectral functions of the last chapter we can now evaluate the dispersion integrals, Eqs. 48-50, in order to obtain the invariant amplitudes in the s-channel. Eq. 53 then provides the (on-shell) baryon-baryon interaction due to correlated $\pi\pi$ and $K\overline{K}$ exchange.

3.5.1 The effective coupling constants

The results will be presented in terms of the effective coupling strengths $G^{\sigma,\rho}_{AB\to CD}(t)$, which have been introduced in Eqs. 54, 56 to parametrize the correlated processes by

(sharp mass) σ and ρ exchange. We stress once more that this parametrization does not involve any approximations as long as the full t-dependence of the effective coupling strengths are taken into account. The parameters of σ resp. ρ exchange (mass of the exchanged particle: m_σ, m_ρ; cutoff mass: Λ_σ, Λ_ρ) are chosen to have the same values in all particle channels. m_σ and m_ρ have been set to the values used in the Bonn-Jülich models of the NN [10] and YN [3] interactions, i.e. $m_\sigma = 550\ MeV$, $m_\rho = 770\ MeV$. The cutoff masses have been chosen such that the coupling strengths in the $S = 0, -1$ baryon-baryon channels vary only weakly with t. The resulting values ($\Lambda_\sigma = 2.8\ GeV$, $\Lambda_\rho = 2.5\ GeV$) are quite large compared to the values of the phenomenological parametrizations used in Refs. [10, 3] and thus represent very hard form factors. Note that, unless stated otherwise, the upper limit t'_{max} in the dispersion integrals is put to $120 m_\pi^2$. Fig. 20 shows the effective coupling strengths $G^\sigma_{AB}(t)$ in the baryon-baryon channels considered here, as function of $-t$. With the exception of $G^\sigma_{\Sigma\Sigma}(t)$ (dash-dotted curve) the effective σ coupling strengths vary only weakly with $-t$, which proves that $550\ MeV$ is a realistic choice for the σ mass. In the $\Sigma\Sigma$ channel the suitable mass lies somewhat higher due to the strong coupling to the $K\overline{K}$ channel generated by Δ exchange, cf. Fig. 11. If we compare the relative strengths of effective σ exchange in the various baryon-baryon channels we observe the same features observed already for the spectral functions: The scalar-isoscalar part of correlated $\pi\pi$ and $K\overline{K}$ exchange is in the NN channel about twice as large as in both YN channels and by a factor 3-4 larger than in the $S = -2$ channels.

For sharp mass σ exchange the following equation holds

$$G^\sigma_{NN\to NN}(t) G^\sigma_{\Sigma\Sigma\to\Sigma\Sigma}(t) = [G^\sigma_{N\Sigma\to N\Sigma}(t)]^2 \quad .$$

In other words, the three processes are determined by two coupling constants $g_{\sigma NN} \equiv \sqrt{G^\sigma_{NN\to NN}}$ and $g_{\sigma\Sigma\Sigma} \equiv \sqrt{G^\sigma_{\Sigma\Sigma\to\Sigma\Sigma}}$, such that $G^\sigma_{N\Sigma\to N\Sigma}(t) = g_{\sigma NN} g_{\sigma\Sigma\Sigma}$. For correlated exchanges this is not necessarily true anymore; here we have in general

$$G^\sigma_{NN\to NN}(t) G^\sigma_{\Sigma\Sigma\to\Sigma\Sigma}(t) \geq [G^\sigma_{N\Sigma\to N\Sigma}(t)]^2 \quad ,$$

which is just a consequence of the Schwarz inequality relation

$$|\int f(t) g(t) dt|^2 \leq \int f(t)^2 dt \times \int g(t)^2 dt \quad .$$

The equality holds if both functions have the same t-dependence. This is roughly fulfilled for NN and $\Lambda\Lambda$, but not for $\Sigma\Sigma$; therefore the equality approximately holds in the first but not in the second case. Consequently, in general, effective vertex couplings for correlated exchanges are not well defined since they might take different values in different baryon-baryon channels.

Tab. 5 contains the coupling strengths at $t = 0$. Besides the results for our full model for correlated $\pi\pi$ and $K\overline{K}$ exchange the table includes results obtained when neglecting the $B\overline{B} \to K\overline{K}$ Born amplitudes. Obviously the inclusion of these amplitudes provides only 15% of the coupling strengths in the NN channel; it is much more important in the channels with strangeness, in fact providing the dominant part in the $S = -2$ channels. Furthermore the table contains results obtained when uncorrelated contributions involving spin-1/2 baryons only are subtracted from the discontinuities of the invariant baryon-baryon amplitudes in order to avoid double counting in case a

simple OBE-model is used in the s-channel. For the full Bonn NN model contributions involving spin-3/2 baryons have to be subtracted too (as done in general in this work) since corresponding contributions are already treated explicitly in the s-channel. Obviously processes involving spin-3/2 baryons increase the 'true' correlated contribution by about 30% in all channels.

In the ρ-channel the spectral functions are dominated by the resonant contributions in the region of the ρ resonance. Therefore the effective coupling strengths ${}^{ij}G^{\rho}_{AB\to CD}(t)$ ($ij = VV, VT, TV, TT$) vary even more slowly with t than those in the σ-channel. Because of this weak t-dependence it is for the moment sufficient to consider only the values of coupling strengths at $t = 0$. They are shown for the NN channel in Tab. 6, for YN in Tab. 7 and in Tab. 8 for the $S = -2$ baryon-baryon channels.

Note that for the equal (unequal) mass case the results are given in terms of 3 (4) coupling strengths. For equal masses the present description in terms of correlated $\pi\pi$ exchange is more involved compared to sharp mass ρ-exchange since the latter can be characterized by two parameters only, the vector and tensor coupling constant. Also, as in the scalar channel, there is no definite relation between coupling strengths in the various channels so that vertex coupling constants cannot be uniquely extracted but depend on the channel chosen. (Thus it is not surprising that our ρNN coupling strengths, which are determined in the NN system, are not fully consistent with the vector and tensor coupling constants derived by Höhler and Pietarinen [18, 19] from πN scattering, though both calculations agree of course qualitatively.)

Tables 6-8 include results obtained when only the ρ-pole terms are considered in the $B\overline{B'} \to \mu\overline{\mu}$ Born amplitudes. In this case the effective coupling strengths are still $SU(3)$ symmetric, i.e. they roughly fulfill the relations

$$\begin{aligned} g_{\Sigma\Lambda\rho} &= 0 \quad , & f_{\Sigma\Lambda\rho} &= \tfrac{2\sqrt{3}}{5}\,(g_{NN\rho} + f_{NN\rho}) \quad , \\ g_{\Sigma\Sigma\rho} &= 2g_{NN\rho} \quad , & f_{\Sigma\Sigma\rho} &= -\tfrac{2}{5}\,(3g_{NN\rho} - 2f_{NN\rho}) \quad , \\ g_{\Xi\Xi\rho} &= g_{NN\rho} \quad , & f_{\Xi\Xi\rho} &= -\tfrac{1}{5}\,(6g_{NN\rho} + f_{NN\rho}) \quad . \end{aligned} \tag{85}$$

with $\alpha_v^e = 1$ and $\alpha_v^m = 0.4$. (We remind the reader that we have chosen the bare couplings to exactly obey $SU(3)$ symmetry.) Obviously the influence of ($SU(3)$ broken) baryon masses on the ρ-pole contributions is small, probably because our calculations are performed in the pseudophysical region, far below the baryon-baryon thresholds. As expected however the effective coupling strengths of the complete calculation do not respect $SU(3)$ symmetry anymore, due to the sizable influence of the (non-pole) baryon-exchange processes.

Doing again a restricted subtraction of spin-1/2 baryon contributions only (suitable for an OBE model) we now do not have a unique trend for the change of coupling strengths in all channels, as found before in the scalar case. In case of the vector coupling strength (VV) in the NN system the restricted subtraction leads even to a smaller value. This is not too surprising if one realizes that the ρ-vector coupling strengths are obtained from differences of approximately equal spectral functions so that well-controlled changes in the spectral functions can lead sometimes to large modifications of coupling strengths, in arbitrary direction.

At this point we would like to make a remark about the sensitivity of our results to the upper limit in the dispersion integral, t'_{max}. It is in fact quite small: Lowering t'_{max} from the generally used value of $120m_\pi^2$ to $80m_\pi^2$ the effective σ-coupling strengths are

increased by less than 10%; in the ρ-channel the variations are even smaller. Moreover, ratios of coupling strengths in the various channels are practically unchanged.

Two points remain to be addressed:

(i) For the couplings of baryons to pseudoscalar mesons we have assumed in our calculations $SU(3)$ symmetry to be realized for pseudoscalar-type coupling, cf. Sect. 2.4.1. Results are different when the same symmetry is assumed for couplings of pseudovector type, since (apart from the $NN\pi$ coupling) $BB'\mu$ couplings are then increased by a factor $(M_B{+}M'_B)/2M_N$ leading to much stronger σ-coupling strengths in the strange baryon-baryon channels, see Table 9.

(ii) We have assumed for the $F/(F+D)$ ratios $\alpha_p = \alpha_v^m = 0.4$ predicted by the static quark model. If we change α_p by about 10% (to 0.45) the resulting changes in the effective σ-coupling strengths are much less than 10%. The reason is that part of the Born amplitudes are increased while others are decreased, so that the total effect is quite small. The situation is completely different for α_v^m, which determines the bare tensor couplings $f^{(0)}_{BB'\rho}$ in the ρ pole graph. The same increase of α_v^m (to 0.45) leads to strong modifications in the ρ coupling strengths, especially for the tensor-tensor part.

3.5.2 Comparison with other models

In the NN channel we can also determine the effective σ- and ρ-coupling strengths from the quasiempirical $N\overline{N} \to \pi\pi$ amplitudes [18, 19]. Corresponding spectral functions have already been discussed before. We now show in Fig. 21 the results for the product of effective coupling strengths and form factors obtained from our microscopic model and the quasiempirical amplitudes [18, 19], in comparison to values used in the (full) Bonn potential [10]. Since the quasiempirical amplitudes are available only up to $t'_{max} = 50m_\pi^2$, a corresponding cutoff is used in the dispersion integral. If we use the same lower cutoff also for our model corresponding results essentially agree with those obtained for the quasiempirical amplitudes. Obviously the slightly stronger increase of the spectral function ρ_S^σ of the microscopic model at small t' roughly compensates for the larger maximum of the quasiempirical spectral function at $t' \approx 20m_\pi^2$. Furthermore we obtain a considerable reduction of strength in the σ-channel from inclusion of the repulsive contributions above $t' = 50m_\pi^2$ whereas in the ρ-channel such pieces have only a very small influence. Especially in the ρ-channel our results are considerably larger than the values used in the Bonn potential. The reason is the form factor with $\Lambda_{NN\rho} = 1.4\,GeV$ used in the Bonn potential, which reduces the strength at $t = 0$ by as much as 50%. One final point remains to be addressed: Our present results differ considerably (by up to 30%) from our former calculations based on a different microscopic model for the $N\overline{N} \to \pi\pi$ amplitudes. The reason for these discrepancies (in the spectral functions and effective coupling strengths) is that the subtraction of the uncorrelated terms from the discontinuities is model-dependent. Both models differ in the parametrization of Δ exchange (75); furthermore the model of Ref. [12] does not include the ρ-pole term in the transition amplitude. Still both models provide a similar description of the quasiempirical data.

The average size of the effective coupling strengths is only a rough measure of the strength of correlated $\pi\pi$ and $K\overline{K}$ exchange in the various particle channels. The

precise energy dependence of the correlated exchange as well as its relative strength in the different partial waves of the s-channel reaction is determined by the spectrum of the exchanged invariant masses, i.e. the spectral functions, leading to a different t-dependence of the effective coupling strengths.

Fig. 22 shows the on-shell NN potentials in spin-singlet states with angular momentum $L = 0, 2,$ and 4, which are generated by the scalar-isoscalar part of correlated $\pi\pi$ and $K\overline{K}$ exchange. As expected it is attractive throughout. Slight differences occur between the potentials derived from the microscopic model for the $B\overline{B'} \rightarrow \mu\overline{\mu}$ amplitudes and those determined from the quasiempirical $N\overline{N} \rightarrow \pi\pi$ amplitudes, which can be traced to differences in the spectral function $\rho^{\sigma}_{S}(t)$ (cf. Fig. 19): For small t' the microscopic input is larger; therefore the corresponding potential in high partial waves (which is dominated by the small-t' behavior) is by about 20% larger than the quasiempirical result. In the $^1S_0$ partial wave, on the other hand, medium and short ranged exchange processes characterized by larger t'-values contribute. In this region the microscopic amplitudes are considerably weaker; furthermore they contain the repulsive contributions above the $K\overline{K}$ threshold (cf. Fig. 16). Consequently the resulting potential is somewhat less attractive in the $^1S_0$ partial wave. In agreement with Ref. [12] our present results (evaluated either from the microscopic model or the quasiempirical amplitudes) are stronger than σ'-exchange of the full Bonn potential. The difference is especially large in high partial waves since σ'-exchange, which corresponds to a spectral function proportional to $\delta(t' - m_{\sigma}^2)$, does not contain the long-range part of the correlated processes. Indeed if we parametrize our results derived from the microscopic model by σ-exchange as before (Sect. 3.5.1) but use for the effective coupling strength $G^{\sigma}_{NN\rightarrow NN}(t)$ the constant value at $t = 0$ we obtain rough agreement with our unapproximated result in the $^1S_0$ partial wave but underestimate it considerably in high partial waves. Obviously the replacement of correlated $\pi\pi$ and $K\overline{K}$ exchanges by an exchange of a sharp mass σ meson with t-independent coupling strength cannot provide a simultaneous description of low and high partial waves.

It is interesting to compare our results for the effective σ- and ρ- coupling strengths in baryon-baryon channels with non-vanishing strangeness with those used in the hyperon-nucleon interaction models of the Nijmegen [4, 5, 6] and Jülich [3] groups. These are (with the exception of the Jülich model B which will not be considered in the following) OBE models, i.e. σ (and ρ) exchange effectively include uncorrelated processes involving the Δ-isobar. Therefore we have to use for comparison dispersion-theoretic results in which only uncorrelated processes involving spin–1/2 particle intermediate states have been subtracted. Table 10 shows the relative coupling strengths in the different baryon-baryon channels for various models, in the σ-channel. Apart from the Nijmegen model D [4], in which the scalar ϵ-meson is treated as an $SU(3)$ singlet and therefore couples with the same strength to all channels, the interaction is by far strongest in the NN channel for all remaining models, and it becomes weaker with increasing strangeness. Obviously, the Nijmegen soft core model [6] is nearest to the dispersion-theoretic predictions. Table 11 shows the analogous results in the ρ-channel for the vector-vector (VV) and tensor-tensor (TT) components. There are sizable differences between the effective coupling constants from correlated exchange and the coupling constants of the OBE-models as well as among the OBE-models themselves. The latter models assume the vector coupling to the isospin current to be universal, which fixes its relative strength in the different particle channels (apart from form factors included in the Jülich model):

In the $N\Sigma$ channel it is twice as large as in the NN channel and vanishes for the transition $N\Lambda \to N\Sigma$. The correlated exchange result deviates strongly, which is another manifestation that $SU(3)$ symmetry does not hold for correlated exchanges, even if it is assumed for the bare $\rho BB'$ couplings present in our microscopic model.

4 Summary and Outlook

An essential part of baryon-baryon interactions is the strong attraction of medium range, which in one-boson-exchange models is parametrized by an exchange of a fictitious scalar-isoscalar meson with a mass of about $500 MeV$. In extended meson exchange models this part is naturally generated by two-pion-exchange processes. Besides uncorrelated processes correlated terms have to be considered in which both pions interact during their exchange; in fact these terms provide the main contribution to the intermediate-range interaction.

In the scalar-isoscalar channel of the $\pi\pi$ interaction the coupling to the $K\overline{K}$ channel plays a strong role, which has to be explicitly included in any model meant to be realistic for energies near and above the $K\overline{K}$ threshold. As kaon exchange is an essential part of hyperon-nucleon interactions a simultaneous investigation of correlated $\pi\pi$ and $K\overline{K}$ exchanges is clearly suggested. In this work we have therefore derived the correlated $\pi\pi$ as well as $K\overline{K}$ exchange contributions in various baryon-baryon channels. Starting point of our calculations was a microscopic model for the transition amplitudes of the baryon-antibaryon system ($B\overline{B'}$) into two pseudoscalar mesons ($\pi\pi$, $K\overline{K}$) for energies below the $B\overline{B'}$ threshold. The correlations between the two mesons have been taken into account by means of $\pi\pi - K\overline{K}$ amplitudes (determined likewise fieldtheoretically [13, 17]), which provide an excellent reproduction of empirical $\pi\pi$ data up to $1.3 GeV$. With the help of unitarity and dispersion-theoretic methods we have then determined the baryon-baryon amplitudes for correlated $\pi\pi$ and $K\overline{K}$ exchange in the $J^P = 0^+$ (σ) and $J^P = 1^-$ (ρ) t-channel.

In the σ-channel the strength of correlated $\pi\pi$ and $K\overline{K}$ exchange decreases with the strangeness of the baryon-baryon channels becoming more negative. In the NN channel the scalar-isoscalar part of correlated exchanges is by about a factor of 2 stronger than in both hyperon-nucleon channels (ΛN, ΣN) and by a factor 3 to 4 stronger than in the $S = -2$ channels ($\Lambda\Lambda$, $\Sigma\Sigma$, $N\Xi$). The influence of $K\overline{K}$ exchange is strong in baryon-baryon channels with non-vanishing strangeness while it is small in the NN channel. This feature can be traced to different coupling constants and isospin factors and especially to the different masses involved in the various baryon-antibaryon channels.

The role of correlated $K\overline{K}$ exchange is small in the ρ-channel. Here the correlations are dominated by the (genuine) ρ-resonance in the $\pi\pi$ interaction. Among the various $B\overline{B'} - \pi\pi, K\overline{K}$ Born amplitudes the direct coupling of the ρ-resonance to the baryons in the form of a ρ-pole graph provides the dominant contribution to correlated exchange.

It turns out that our results depend only slightly on the upper limit (cutoff) introduced in the dispersion integral. Some uncertainty results from applying $SU(3)$ resp. $SU(6)$ relations to either pseudoscalar or pseudovector π and K coupling constants. Note that the same problem occurs already in OBE-models of the hyperon-nucleon interaction. Ultimately it has to be decided by comparison with experiment which procedure is to be preferred. Moreover, if instead of $SU(6)$ symmetry $SU(3)$ symmetry is assumed only, the results for correlated exchanges depends on the $F/(F + D)$ ratios α_p

and α_v^m. While the dependence on α_p is only weak variation ofα_v^m leads to noticeable changes in the model predictions for the correlated exchange in the ρ-channel. Also here a final decision about the correct choice of α_p and α_v^m can be made only by comparison with experiment. Again these parameters occur already in OBE hyperon-nucleon models. Therefore no new parameters are introduced when including correlated $\pi\pi$ and $K\overline{K}$ exchange in baryon-baryon interaction models. On the contrary, the elimination of single σ and ρ exchange reduces the number of free parameters and thus enhances the predictive power of corresponding interaction models.

Our results can be represented in terms of suitably defined effective coupling strengths. It turns out that the resulting values in the various baryon-baryon channels are not connected by $SU(3)$ relations. For example, although we have even assumed $SU(6)$ symmetry for the coupling strength of the bare ρ to the baryon current sizable baryon exchange processes destroy this symmetry in the final effective couplings. Consequently the assumption of $SU(3)$ symmetry for single σ- and ρ-exchange is not supported by our findings.

With this model constructed in the present work it is now possible to take correlated $\pi\pi$ and $K\overline{K}$ exchange reliably into account in the various baryon-baryon channels. Especially in channels in which only little empirical information exists the elimination of phenomenological σ- and ρ-exchange considerably enhances the predictive power of baryon-baryon interaction models. Clearly the inclusion of correlated exchange in existing interaction models (e.g. the Bonn NN potential [10] and the Jülich YN models [3]) requires readjustment of free model parameters to the empirical data. Having fixed these parameters in the NN and YN channel the interaction model can then be extended parameter-free to other baryon-baryon channels with strangeness $S = -2$ using $SU(3)$ arguments for the genuine couplings. In this way in the frame of the Bonn-Jülich models the possibility arises for the first time to make sensible statements about the existence of bound baryon-baryon states with strangeness $S = -2$, which should be of some importance regarding the analysis of H-dibaryon experiments.

Appendix

A Conventions

As far as possible the conventions in this work are chosen in accordance with Ref. [34]. The helicity spinors $u(\vec{p},\lambda)$ ($v(\vec{p},\lambda)$) of a Dirac particle of spin 1/2 and mass M are solutions of the free Dirac equation in momentum space for positive (negative) energy and helicity λ

$$\begin{aligned} (\not{p} - M)u(\vec{p},\lambda) &= 0 \quad , \\ (\not{p} + M)v(\vec{p},\lambda) &= 0 \quad , \end{aligned} \tag{86}$$

with the 4-momentum p^μ ($p^0 \equiv E_p = +\sqrt{M^2 + \vec{p}^{\,2}}$) and $\not{p} \equiv p^\mu \gamma_\mu$. With the phase convention of Ref. [36] the helicity spinors read ($\lambda = \pm 1/2$)

$$u(\vec{p},\lambda) = \sqrt{\frac{\epsilon_p}{2M}} \begin{pmatrix} 1 \\ 2\lambda|\vec{p}|/\epsilon_p \end{pmatrix} |\,\lambda\,\rangle \quad ,$$

$$v(\vec{p},\lambda) \;=\; \sqrt{\frac{\epsilon_p}{2M}}\begin{pmatrix} -|\vec{p}|/\epsilon_p \\ 2\lambda \end{pmatrix} |-\lambda\,\rangle \quad , \tag{87}$$

where $\epsilon_p \equiv M + E_p$ They are normalized according to

$$\begin{aligned} \overline{u}(\vec{p},\lambda)u(\vec{p},\lambda') &= \delta_{\lambda\lambda'} \quad , \\ \overline{v}(\vec{p},\lambda)v(\vec{p},\lambda') &= -\delta_{\lambda\lambda'} \quad , \end{aligned} \tag{88}$$

with $\overline{u}(\vec{p},\lambda) = u^\dagger(\vec{p},\lambda)\gamma_0$.

If $\vec{p}$ lies in the xz-plane and encloses the polar angle θ with the $\check{e}_z$-axis the eigenstates $|\,\lambda\,\rangle$ of the helicity operator,

$$\frac{\vec{\sigma}}{2}\cdot\frac{\vec{p}}{|\vec{p}|}\,|\,\lambda\,\rangle = \lambda\,|\,\lambda\,\rangle \quad , \tag{89}$$

are related to the Pauli spinors χ_m by

$$|\,\lambda\,\rangle = \exp\left(-\frac{i}{2}\sigma_2\theta\right)\chi_\lambda \quad . \tag{90}$$

where

$$\vec{\sigma}\cdot\check{e}_z\;\chi_{\pm\frac{1}{2}} = \pm\chi_{\pm\frac{1}{2}} \quad . \tag{91}$$

The same phase convention as in Ref. [36] is adopted for the helicity spinors 87; namely, the particle and antiparticle spinors are related by the charge conjugation $\mathcal{C}$ [34]:

$$v(\vec{p},\lambda) = \mathcal{C}\overline{u}^T(\vec{p},\lambda) = i\gamma_2 u^*(\vec{p},\lambda) \quad . \tag{92}$$

Helicity eigenstates of two-particle systems in the center-of-mass frame are built as a product of two helicity spinors according to the phase convention of Jacob and Wick [20]. In case of two spin-1/2 particles 1 and 2, the helicity spinors 87 are used for particle 1 (momentum $\vec{p}$) and for particle 2 (momentum $-\vec{p}$) the following spinors are used:

$$u(-\vec{p},\lambda_2) \;=\; \sqrt{\frac{\epsilon_p}{2M}}\begin{pmatrix} 1 \\ 2\lambda_2|\vec{p}|/\epsilon_p \end{pmatrix} |\,\lambda_2\,\rangle \quad , \tag{93}$$

$$v(-\vec{p},\lambda_2) \;=\; \sqrt{\frac{\epsilon_p}{2M}}\begin{pmatrix} |\vec{p}|/\epsilon_p \\ -2\lambda_2 \end{pmatrix} |-\lambda_2\,\rangle \quad , \tag{94}$$

where

$$|\,\lambda_2\,\rangle = \exp\left(-\frac{i}{2}\sigma_2\theta\right)\chi_{-\lambda_2} \quad . \tag{95}$$

The hadronic field operators can be expanded in momentum space solutions of the corresponding equation of motion. For Dirac particles ($J^P = \frac{1}{2}^+$) and (pseudo)scalar particles ($J^P = 0^\pm$), see Ref. [34]. For spin-1 and spin-3/2 particles the field operators ϕ^μ and ψ^μ, respectively, then read:

$$\phi^\mu(x) = \frac{1}{(2\pi)^{3/2}}\sum_\lambda\int d^3k\frac{1}{\sqrt{2\omega_k}}\epsilon^\mu(\vec{k},\lambda)\left[a(\vec{k},\lambda)e^{-ik\cdot x} + a^\dagger(\vec{k},\lambda)e^{ik\cdot x}\right] \tag{96}$$

$$\psi^\mu(x) = \frac{1}{(2\pi)^{3/2}}\sum_\Lambda\int d^3p\sqrt{\frac{M}{E_p}}\left[b(\vec{p},\Lambda)u^\mu(\vec{p},\Lambda)e^{-ip\cdot x} + d^\dagger(\vec{p},\Lambda)v^\mu(\vec{p},\Lambda)e^{ip\cdot x}\right] \tag{97}$$

with the polarization vector $\epsilon^\mu(\vec{k},\lambda)$, cf. [37]. Note that the hermitian conjugated component of the spherical operator a is related to the component of the hermitian operator $a^\dagger$ by [38]:

$$a(\vec{k},\lambda)^\dagger = (-1)^\lambda a^\dagger(\vec{k},-\lambda) \quad . \tag{98}$$

The Rarita-Schwinger spinors [37] $u^\mu(\vec{p},\Lambda)$ and $v^\mu(\vec{p},\Lambda)$ are solutions of the Rarita-Schwinger equation

$$\begin{array}{lrcl@{\qquad}rcl} & (\not{p}-M)u^\mu(\vec{p},\Lambda) &=& 0, & (\not{p}+M)v^\mu(\vec{p},\Lambda) &=& 0 \\ \text{and} & & & & & & \\ & \gamma_\mu u^\mu(\vec{p},\Lambda) &=& 0, & \gamma_\mu v^\mu(\vec{p},\Lambda) &=& 0. \end{array} \tag{99}$$

The creation and destruction operators of bosons (fermions) follow the usual (anti-)commutator relations.

For spin-1 and spin-3/2 particles [32] the Feynman propagators $S_v(k)$ and $S_D^{\mu\nu}(p;A)$, which are derived from the time-ordered product of the corresponding field operators [34], read in momentum space:

$$S_v(k) = \frac{-g^{\mu\nu} + k^\mu k^\nu/m^2}{k^2 - m^2 + i\epsilon} \tag{100}$$

$$\begin{aligned} S_D^{\mu\nu}(p;A) &= \frac{\not{p}+M}{p^2-M^2+i\epsilon}\left[-g^{\mu\nu} + \frac{\gamma^\mu\gamma^\nu}{3} + \frac{2}{3M^2}p^\mu p^\nu - \frac{p^\mu\gamma^\nu - p^\nu\gamma^\mu}{3M}\right] \\ &\quad + S^{\mu\nu}_{\mathrm{non-pole}}(p;A) \end{aligned} \tag{101}$$

with the non-pole part

$$S^{\mu\nu}_{\mathrm{non-pole}}(p;A) = \frac{1}{3M^2}\frac{A+1}{2A+1}\left[\gamma^\mu \frac{\frac{(A+1)}{2}\not{p} + AM}{2A+1}\gamma^\nu - (p^\mu\gamma^\nu + p^\nu\gamma^\mu)\right] \quad . \tag{102}$$

Without restricting the general validity of the results the parameter A is set in our work to $A=-1$ so that the non-pole part vanishes (see Sect. 2.4.1).

B Matrix elements of $B\overline{B'} \to \mu\overline{\mu}$ Born amplitudes

In order to evaluate the helicity amplitudes 77 of the $B\overline{B'} \to \mu\overline{\mu}$ ($\mu\overline{\mu} = \pi\pi,\ K\overline{K}$) Born terms 74-76 the coordinate system is conveniently chosen such that the relative momentum $\vec{q}$ of the $B\overline{B'}$ state points along the z-axis and the relative momentum $\vec{k}$ of the two pseudoscalar mesons $\mu\overline{\mu}$ lies in the xz-plane; i.e., the components of the two momenta read

$$\vec{q} = \begin{pmatrix} 0 \\ 0 \\ q \end{pmatrix} \quad , \qquad \vec{k} = \begin{pmatrix} k\sin\vartheta \\ 0 \\ k\cos\vartheta \end{pmatrix} \quad , \tag{103}$$

with the scattering angle $\vartheta = \sphericalangle(\vec{p},\vec{k})$.

As explained in Sect. 2.4 the $B\overline{B'} \to \mu\overline{\mu}$ amplitudes need to be evaluated only for $B\overline{B'}$ states being on their mass-shell:

$$\sqrt{t} = E_B + E_{B'} = \sqrt{M_B + q^2} + \sqrt{M_{B'} + q^2} \tag{104}$$

Therefore, in the following, t is suppressed as an argument of the helicity amplitudes.

In case of the $B\overline{B'} \to \pi\pi$ amplitudes one has in principle to take into account also the exchange graph (with the external pion lines exchanged) arising from the symmetrized $\pi\pi$ states. However, this can be simply done by considering the selection rule for the $\pi\pi$ states: $(-1)^{J+I} = +1$ (J: total angular momentum, I: total isospin); i.e., multiplying the direct, partial wave decomposed $B\overline{B'} \to \pi\pi$ amplitude by a factor $(1 + (-1)^{J+I})$.

In the final results for the $B\overline{B'} \to \mu\overline{\mu}$ Born amplitudes given below we introduced the following abbreviations:

$$\begin{aligned} \alpha_+ &\equiv \epsilon_B + \epsilon_{B'} \ , \qquad & \alpha_- &\equiv \epsilon_B - \epsilon_{B'} \ , \\ \beta_+ &\equiv \epsilon_B \epsilon_{B'} + q^2 \ , \qquad & \beta_- &\equiv \epsilon_B \epsilon_{B'} - q^2 \ , \end{aligned} \tag{105}$$

with $\epsilon_B \equiv M_B + E_B$.

B.1 Exchange of a $J^P = \frac{1}{2}^+$ baryon

$$\begin{aligned} \langle\, \mu\overline{\mu}, \vec{k}\,|\, V_X \,|\, B\overline{B'}, \vec{q} + + \rangle \;=\; & C\{-q[M_X(t+4k^2)\alpha_+ - q_0(t-4k^2)\alpha_- + (t+4k^2)\beta_-] \\ & +k\cos\theta[4M_X\sqrt{t}\beta_+ + (t-4k^2)\beta_-] \\ & +8k^2 q\cos^2\theta\beta_-\} \quad , \end{aligned} \tag{106}$$

$$\langle\, \mu\overline{\mu}, \vec{k}\,|V_X|\, B\overline{B'}, \vec{q}+-\,\rangle = -Ck\sin\theta\{-4q^2\sqrt{t}\alpha_+ + (t-4k^2)\beta_+ + 4M_X\sqrt{t}\beta_- + 8kq\cos\theta\beta_+\} \ , \tag{107}$$

where

$$C := -\frac{f_{BX\mu} f_{B'X\mu}}{8m_\mu^2\sqrt{\epsilon_B\epsilon_{B'}M_B M_{B'}}}\frac{F_X^2(p^2)}{q_0^2 - E_X^2} \quad , \tag{108}$$

with $E_X = \sqrt{\vec{p}^{\,2} + M_X^2}$ with $\vec{p} = \vec{q} - \vec{k}$.

B.2 Exchange of a $J^P = \frac{3}{2}^+$ baryon

$$
\begin{aligned}
&\langle\, \mu\overline{\mu}, \vec{k} \,|\, V_X \,|\, B\overline{B'}, \vec{q} + + \,\rangle = \\
&C \Bigg\{ \tfrac{1}{q_0^2 - E_X^2} \{ \quad q \quad [M_X A(t,k)\alpha_+ + q_0 A(t,k)\alpha_- + 2M_X k^2 \sqrt{t}\beta_+ + A(t,k)\beta_-] \\
&\qquad + k\cos\theta \quad [8M_X q^2 k^2 \alpha_+ + 8q_0 q^2 k^2 \alpha_- \\
&\qquad\qquad + 2M_X\sqrt{t}(q_0^2 - M_X^2 - q^2 - k^2)\beta_+ + (8k^2q^2 - A(t,k))\beta_-] \\
&\qquad + 2k^2 q\cos^2\theta \quad [-2M_X q^2 \alpha_+ - 2q_0 q^2 \alpha_- + M_X\sqrt{t}\beta_+ - 2(q^2 + 2k^2)\beta_-] \\
&\qquad + 4k^3 q^2 \cos^3\theta \quad \beta_- \} \\
&\quad + x_\Delta \{ \quad -q \quad [M_X(t + 4k^2)\alpha_+ - 2q_0 t\alpha_-] \\
&\qquad + 4k\cos\theta \quad [M_X\sqrt{t}\beta_+ - 2k^2\beta_-] \\
&\qquad + 8k^2 q\cos^2\theta \quad \beta_- \} \\
&\quad + x_\Delta^2 \{ \quad -q \quad [2M_X(t + 4k^2)\alpha_+ - q_0(t - 4k^2)\alpha_- + (t + 4k^2)\beta_-] \\
&\qquad + k\cos\theta \quad [+8M_X\sqrt{t}\beta_+ + (t - 4k^2)\beta_-] \\
&\qquad + 8k^2 q\cos^2\theta \quad \beta_- \} \Bigg\}
\end{aligned}
\tag{109}
$$

$$
\begin{aligned}
&\langle\, \mu\overline{\mu}, \vec{k} \,|\, V_X \,|\, B\overline{B'}, \vec{q} + - \,\rangle = \\
&Ck\sin\theta \times \Bigg\{ \tfrac{1}{q_0^2 - E_X^2} \{ \quad - \quad [2M_X^2\sqrt{t}q^2\alpha_+ + 2M_X q_0 q^2 \sqrt{t}\alpha_- \\
&\qquad\qquad - A(t,k)\beta_+ + 2M_X\sqrt{t}(q_0^2 - M_X^2 - k^2)\beta_-] \\
&\qquad + 2kq\cos\theta \quad [4k^2\beta_+ - M_X\sqrt{t}\beta_-] \\
&\qquad - 4k^2q^2\cos^2\theta \quad \beta_+ \} \\
&\quad + 4x_\Delta \{ \quad 2k^2\beta_+ - M_X\sqrt{t}\beta_- \\
&\qquad - 2kq\cos\theta \quad \beta_+ \} \\
&\quad + x_\Delta^2 \{ \quad - \quad [-4q^2\sqrt{t}\alpha_+ + (t - 4k^2)\beta_+ + 8M_X\sqrt{t}\beta_-] \\
&\qquad - 8kq\cos\theta \quad \beta_+ \} \Bigg\} \quad ,
\end{aligned}
\tag{110}
$$

with

$$
C := -\frac{f_{BX\mu} f_{B'X\mu}}{12 m_\mu^2 M_X^2 \sqrt{\epsilon_B \epsilon_{B'} M_B M_{B'}}} F_X^2(p^2) \quad , \tag{111}
$$

$$
A(t,k) := q_0^2 t - 4k^2 M_X^2 - M_X^2 t - 4k^4 \quad . \tag{112}
$$

B.3 ρ-pole graph

For the ρ-pole graph the ρ-propagator and the form factor $F_{\mu\mu\rho}(k^2)$ (cf. Eq. 72) do not depend on the scattering angle θ. Therefore, the partial wave decomposition can easily be performed analytically. The results are:

$$
\langle\, \mu\overline{\mu}, k \,|\, V_\rho^J \,|\, B\overline{B'}, q + + \,\rangle \;=\; \delta_{J1} C_\rho k [g^{(0)}_{BB'\rho}\beta_- + \frac{f^{(0)}_{BB'\rho}}{2M_N}\sqrt{t}\beta_+], \tag{113}
$$

$$\langle\,\mu\overline{\mu},k\,|\,V_\rho^J\,|\,B\overline{B'},q+-\,\rangle \;=\; \delta_{J1}\sqrt{2}C_\rho k[g^{(0)}_{BB'\rho}\beta_+ + \frac{f^{(0)}_{BB'\rho}}{2M_N}\sqrt{t}\beta_-], \tag{114}$$

where

$$C_\rho := \frac{4\pi}{3}\frac{g^{(0)}_{\mu\mu\rho}}{\sqrt{\epsilon_B\epsilon_{B'}M_BM_{B'}}}\frac{F_{\mu\mu\rho}(k^2)}{t-(m^{(0)}_\rho)^2} \quad . \tag{115}$$

C $SU(3)$ relations for coupling constants

The microscopic model for the $B\overline{B'} \to \pi\pi, K\overline{K}$ amplitudes presented in Sect. 2.4 imposes $SU(3)$ symmetry to the coupling constants at the hadronic vertices in order to keep the number of free parameters as low as possible. The $SU(3)$ relations for the coupling constants are derived by constructing an $SU(3)$-invariant interaction Lagrangian from particle field operators, which possess a well-defined behavior under $SU(3)$ transformations. This method is well-established [39] and makes use of the $SU(3)$ Clebsch-Gordan coefficients and the so-called isoscalar factors which are tabulated in Ref. [39]. Therefore, we give in the following only the final results relevant for this work.

The coupling of the pseudoscalar meson octet (π, η_8, K, $\overline{K}$) to the current of the $J^P = 1/2^+$ baryon octet (N, Λ, Σ, Ξ) is described by the Lagrangian [39]

$$\begin{aligned}
\mathcal{L}_{B'Bp} \;=\;& g_{NN\pi}(N^\dagger\vec{\tau}N)\cdot\vec{\pi} && + \; g_{NN\eta_8}(N^\dagger N)\eta_8 \\
+\;& g_{\Lambda NK}\left[(N^\dagger K)\Lambda + \Lambda^\dagger(K^\dagger N)\right] && + \; g_{\Sigma NK}\left[(N^\dagger\vec{\tau}K)\cdot\vec{\Sigma} + \vec{\Sigma}^\dagger\cdot(K^\dagger\vec{\tau}N)\right] \\
+\;& g_{\Sigma\Lambda\pi}\left[\vec{\Sigma}^\dagger\cdot\vec{\pi}\Lambda + \Lambda^\dagger\vec{\Sigma}\cdot\vec{\pi}\right] && - \; ig_{\Sigma\Sigma\pi}\left(\vec{\Sigma}^\dagger\times\vec{\Sigma}\right)\cdot\vec{\pi} \\
+\;& g_{\Lambda\Lambda\eta_8}\Lambda^\dagger\Lambda\eta_8 && + \; g_{\Sigma\Sigma\eta_8}\vec{\Sigma}^\dagger\cdot\vec{\Sigma}\eta_8 \\
+\;& g_{\Xi\Lambda K}\left[\Lambda^\dagger(\overline{K}^\dagger\Xi) + (\Xi^\dagger\overline{K})\Lambda\right] && + \; g_{\Xi\Sigma K}\left[\vec{\Sigma}^\dagger\cdot(\overline{K}^\dagger\vec{\tau}\,\Xi) + (\Xi^\dagger\vec{\tau}\overline{K})\cdot\vec{\Sigma}\right] \\
+\;& g_{\Xi\Xi\pi}(\Xi^\dagger\vec{\tau}\,\Xi)\cdot\vec{\pi} && + \; g_{\Xi\Xi\eta_8}(\Xi^\dagger\Xi)\eta_8 \quad ,
\end{aligned} \tag{116}$$

where the coupling constants being of the pion and the kaon are given by

$$\begin{aligned}
& g_{NN\pi} = g \quad , && g_{\Lambda NK} = -\tfrac{1}{\sqrt{3}}g(1+2\alpha) \quad , \\
& g_{\Sigma NK} = g(1-2\alpha) \quad , && g_{\Sigma\Lambda\pi} = \tfrac{2}{\sqrt{3}}g(1-\alpha) \quad , \\
& g_{\Sigma\Sigma\pi} = 2g\alpha \quad , g_{\Xi\Lambda K} = \tfrac{1}{\sqrt{3}}g(4\alpha-1) \quad , && \\
& g_{\Xi\Sigma K} = -g \quad , g_{\Xi\Xi\pi} = -g(1-2\alpha) \quad . &&
\end{aligned} \tag{117}$$

Hence, the coupling of π and K to the baryon octet is determined by two parameters: the coupling strength g_8 and the so-called $F/(F+D)$-ratio α_p.

The transition from pseudoscalar to vector mesons can be simply made by the replacement

$$\pi \to \rho, \quad K \to K^*, \quad \eta \to \phi, \quad \eta' \to \omega. \tag{118}$$

However, as can be seen from the spin-momentum part of the interaction Lagrangian Eq. 41 the vector mesons couple to the baryon octet in two different ways: via the vector and the tensor coupling with coupling constants g and f, respectively. Now, it is not clear which coupling constants underly $SU(3)$ symmetry relations. Besides the canonic

assumption [4, 6] that the $SU(3)$ relations apply to g and f, sometimes [5, 3] the electric (g) and the magnetic ($G = g + f$) coupling constants are subject to $SU(3)$. In fact, both assumptions are in a certain sense equivalent [40]. For given $SU(3)$ parameters (g_1, g_8, α_v^g) and (f_1, f_8, α_v^f) the parameters (G_1, G_8, α_v^m) of the magnetic coupling can be chosen such that both coupling schemes lead to the same tensor couplings $f = G - g$.

Extending the $SU(3)$ symmetry to $SU(6)$, predictions for the $F/(F+D)$-ratios of the different multiplet couplings can be derived [41]:

$$\alpha_p = 0.4, \qquad \alpha_v^e = 1, \qquad \alpha_v^m = 0.4 \quad . \tag{119}$$

$\alpha_v^e = 1$ corresponds to the usual assumption of universal electric coupling of the ρ meson to the isospin current [42], demanding for instance the equality $g_{NN\rho}$ and $g_{KK\rho}$.

The coupling of the pseudoscalar meson octet to the current for the transition between the $J^P = 3/2^+$ baryon decuplet (Δ, Y^*, Ξ^*, Ω) and the $J^P = 1/2^+$ baryon octet is given by the Lagrangian [43]:

$$\begin{array}{rlll}
\mathcal{L}_{DBp} = & f_{N\Delta\pi}(N^\dagger \vec{T}\Delta)\cdot\vec{\pi} & & \\
+ & f_{NY^*K}(N^\dagger\vec{\tau}K)\cdot\vec{Y}^* & + \ f_{\Sigma\Delta K}\vec{\Sigma}^\dagger\cdot(K^\dagger\vec{T}\Delta) & \\
+ & f_{\Lambda Y^*\pi}\Lambda^\dagger\vec{Y}^*\cdot\vec{\pi} & - \ i f_{\Sigma Y^*\pi}\left(\vec{\Sigma}^\dagger\times\vec{Y}^*\right)\cdot\vec{\pi} & + \ f_{\Sigma Y^*\eta_8}\vec{\Sigma}^\dagger\cdot\vec{Y}^*\eta_8 \\
+ & f_{\Lambda\Xi^*K}\Lambda^\dagger(\overline{K}^\dagger\Xi^*) & + \ f_{\Sigma\Xi^*K}\vec{\Sigma}^\dagger\cdot(\overline{K}^\dagger\vec{\tau}\Xi^*) & + \ f_{\Xi Y^*K}(\Xi^\dagger\vec{\tau}\overline{K})\cdot\vec{Y}^* \\
+ & f_{\Xi\Xi^*\pi}(\Xi^\dagger\vec{\tau}\Xi^*)\cdot\vec{\pi} & + \ f_{\Xi\Xi^*\eta_8}(\Xi^\dagger\Xi^*)\eta_8 & \\
+ & f_{\Xi\Omega K}(\Xi^\dagger K)\Omega & & \\
+ & \quad h.c. & &
\end{array} \tag{120}$$

with the π and K coupling constants

$$\begin{array}{lll}
f_{N\Delta\pi} = f \quad , & & \\
f_{NY^*K} = -f/\sqrt{6} \quad , & f_{\Sigma\Delta K} = -f \quad , & \\
f_{\Lambda Y^*\pi} = f/\sqrt{2} \quad , & f_{\Sigma Y^*\pi} = -f/\sqrt{6} \quad , & f_{\Lambda\Xi^*K} = f/\sqrt{2} \quad , \\
f_{\Sigma\Xi^*K} = f/\sqrt{6} \quad , & f_{\Xi Y^*K} = -f/\sqrt{6} \quad , & f_{\Xi\Xi^*\pi} = -f/\sqrt{6} \quad , \\
f_{\Xi\Omega K} = f \quad . & &
\end{array} \tag{121}$$

Obviously, the coupling constants now depend only on one parameter, the coupling strength f.

The $SU(3)$ part of the transition amplitudes is obtained by evaluating the matrix elements of the appropriate products of the $SU(3)$-Lagrangians 116 and 120. If the transition amplitudes are evaluated between states of definite total isospin the results for the $SU(3)$ part can be separated into the product of the corresponding coupling constants and the so-called isospin factor. For instance, the isospin factor $T^X_{B_1\overline{B}_2\to\mu\mu'}(I)$ for the transition of a baryon-antibaryon state $B_1\overline{B}_2$ (with $B_1, B_2 \in \{8_B\}$) to a pseudoscalar meson μ and its antiparticle $\overline{\mu}$ follows from

$$\begin{aligned}
T^X_{B_1\overline{B}_2\to\mu\mu'}(I) g_{B_1X\mu} g_{B_2X\mu'} &\equiv \langle\, \mu\overline{\mu}, Im \,|\; (\mathcal{L}\mathcal{L}')_X \;|\, B_1\overline{B}_2, Im \,\rangle \\
&= \sum_{\substack{m_{B_1}, m_{B_2} \\ m_\mu, m_{\overline{\mu}}}} \langle\, I_{B_1} I_{\bar{B}_2} m_{B_1} m_{\bar{B}_2} \,|\, Im \,\rangle \langle\, I_\mu I_{\overline{\mu}} m_\mu m_{\overline{\mu}} \,|\, Im \,\rangle
\end{aligned}$$

$$\langle\, \mu I_\mu m_\mu, \overline{\mu} I_{\overline{\mu}} m_{\overline{\mu}} \,|\; (\mathcal{L}\mathcal{L}')_X \;|\, B_1 I_{B_1} m_{B_1}, \overline{B}_2 I_{\bar{B}_2} m_{\bar{B}_2} \,\rangle \quad , \tag{122}$$

with m denoting the z-component of the total isospin I and a corresponding notation for the particle isospins $I_{B_1}, I_{\bar{B}_2}, I_\mu, I_{\mu'}$. The index X in $(\mathcal{L}\mathcal{L}')_X$ indicates that the two Lagrangians $\mathcal{L}$ and $\mathcal{L}'$ are coupled by contracting the field operators of the exchanged baryon isomultiplet X (or of the ρ-meson in case of the ρ-pole diagram).

The isospin factors obtained in this way for the processes included in the Born terms of the $B\overline{B'} \to \mu\overline{\mu}$ model (cf. Fig. 7) are listed in Tab. 3. Note that due to the phase convention 61 introduced in connection with the isospin-crossing matrix in Section 2.3.1 some isospin factors (e.g. those for $N\overline{N} \to \pi\pi$) differ in sign from the ones usually used in the literature [12, 14].

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A	B	C	D	I_s	I_t	$X(I_s, I_t)$	F_σ	F_ρ
N	N	N	N	$0, 1$	$0, 1$	$\begin{pmatrix} 1/2 & -3/2 \\ 1/2 & 1/2 \end{pmatrix}$	$1/2$	$1/2$
N	Λ	N	Λ	$1/2$	0	$-1/\sqrt{2}$	$-1/\sqrt{2}$	—
N	Σ	N	Σ	$1/2, 3/2$	$0, 1$	$\begin{pmatrix} 1/\sqrt{6} & -1 \\ 1/\sqrt{6} & 1/2 \end{pmatrix}$	$1/\sqrt{6}$	$1/2$
N	Σ	N	Λ	$1/2$	1	$-\sqrt{3/2}$	—	$-1/\sqrt{2}$
Λ	Λ	Λ	Λ	0	0	1	1	—
Σ	Σ	Σ	Σ	$0, 1, 2$	$0, 1, 2$	$\begin{pmatrix} 1/3 & -1 & 5/3 \\ 1/3 & -1/2 & -5/6 \\ 1/3 & +1/2 & 1/6 \end{pmatrix}$	$1/3$	$1/2$
N	Ξ	N	Ξ	$0, 1$	$0, 1$	$\begin{pmatrix} -1/2 & 3/2 \\ -1/2 & -1/2 \end{pmatrix}$	$-1/2$	$-1/2$

Table 1: Isospin-crossing matrices $X(I_s, I_t)$ for the various baryon-baryon channels $A + B \to C + D$ considered in this paper. The t-channel reaction related by crossing reads $A + \overline{C} \to D + \overline{B}$. In the matrix $X(I_s, I_t)$ I_s increases from top to bottom and I_t from left to right. The last two columns contain the factors F_σ and F_ρ (see text for further explanation).

particle	mass (MeV)	B	S	J^P	I
π	139.57	0	0	0^-	1
K	495.82	0	+1	0^-	$\frac{1}{2}$
ρ	770.	0	0	1^-	1
	$(m_\rho^{(0)}$: 1151.26)				
N	938.919	1	0	$\frac{1}{2}^+$	$\frac{1}{2}$
Λ	1115.68	1	-1	$\frac{1}{2}^+$	0
Σ	1193.1	1	-1	$\frac{1}{2}^+$	1
Ξ	1318.1	1	-2	$\frac{1}{2}^+$	$\frac{1}{2}$
Δ	1232.	1	0	$\frac{3}{2}^+$	$\frac{3}{2}$
$Y^* \equiv \Sigma(1385)$	1385.	1	-1	$\frac{3}{2}^+$	1
Ξ^*	1533.4	1	-2	$\frac{3}{2}^+$	$\frac{1}{2}$
Ω^-	1672.5	1	-3	$\frac{3}{2}^+$	0

Table 2: Particles considered in the model for the $B\overline{B'} \to \mu\overline{\mu}$ Born amplitudes. Given are their masses and the relevant quantum numbers (B: baryon number, S: strangeness, J: spin, P: parity, I: isospin).

$B\overline{B'} \to \mu\overline{\mu}$	process	$T(0)$	$T(1)$	$T(2)$
$N\overline{N} \to \pi\pi$	N	$+\sqrt{6}$	$+2$	
	Δ	$+\sqrt{8/3}$	$-2/3$	
	ρ		-2	
$N\overline{N} \to K\overline{K}$	Λ	$+1$	$+1$	
	Σ, Y^*	$+3$	-1	
	ρ		-2	
$\Lambda\overline{\Lambda} \to \pi\pi$	Σ, Y^*	$-\sqrt{3}$		
$\Lambda\overline{\Lambda} \to K\overline{K}$	N	$-\sqrt{2}$		
	Ξ, Ξ^*	$-\sqrt{2}$		
$\Sigma\overline{\Sigma} \to \pi\pi$	Λ	$+1$	$+1$	$+1$
	Σ, Y^*	$+2$	$+1$	-1
	ρ		-2	
$\Sigma\overline{\Sigma} \to K\overline{K}$	N	$+\sqrt{6}$	-2	
	Δ	$+\sqrt{8/3}$	$+2/3$	
	Ξ, Ξ^*	$+\sqrt{6}$	$+2$	
	ρ		-2	
$\Lambda\overline{\Sigma} \to \pi\pi$	Σ, Y^*		$-\sqrt{2}$	
	ρ		$+\sqrt{2}$	
$\Lambda\overline{\Sigma} \to K\overline{K}$	N		$-\sqrt{2}$	
	Ξ, Ξ^*		$+\sqrt{2}$	
	ρ		$+\sqrt{2}$	
$\Xi\overline{\Xi} \to \pi\pi$	Ξ, Ξ^*	$-\sqrt{6}$	-2	
	ρ		$+2$	
$\Xi\overline{\Xi} \to K\overline{K}$	Λ	-1	$+1$	
	Σ, Y^*	-3	-1	
	Ω^-	-1	-1	
	ρ		$+2$	

Table 3: Isospin factors $T(I)$ for the various $B\overline{B'} \to \mu\overline{\mu}$ Born amplitudes. In the column labelled 'process' the exchanged baryon ($Y^* \equiv \Sigma(1385)$) or a 'ρ' in case of a ρ-pole diagram is given.

$\kappa_\rho^{(0)}$	4.21	
x_Δ	-0.823	
Λ_8	1779.1	MeV
Λ_{10}	1704.7	MeV
α_p	2/5	($SU(6)$[41])
α_v^m	2/5	($SU(6)$[41])

Table 4: Parameter of the microscopic model for the $B\overline{B'} \to \mu\overline{\mu}$ Born amplitudes. The four parameter $\kappa_\rho^{(0)}$, x_Δ, Λ_8 and Λ_{10} are adjusted to the quasiempirical $N\overline{N} \to \pi\pi$ amplitudes of Refs. [18, 19].

$G^\sigma_{AB\to AB}/4\pi$						
	NN	$N\Lambda$	$N\Sigma$	$\Lambda\Lambda$	$\Sigma\Sigma$	$N\Xi$
full model	5.87	2.82	2.58	1.52	1.72	1.19
without $K\overline{K}$ Born terms	5.07	1.80	1.06	0.64	0.22	0.35
subtractions for OBE model	7.77	3.81	3.15	2.00	2.31	1.52

Table 5: Effective σ coupling strengths $G^\sigma_{AB\to AB}(t=0)$ for correlated $\pi\pi$ and $K\overline{K}$ exchange in the various baryon-baryon channels. (The meaning of the several rows is given in the text.)

${}^{ij}G^\rho_{NN\to NN}/4\pi \quad (ij = VV, VT, TV, TT)$			
	NN		
	VV	VT, TV	TT
full model	1.00	5.35	28.91
without baryon exchange	0.52	2.17	9.13
subtraction for OBE model	0.33	5.58	35.23

Table 6: Effective ρ coupling strengths ${}^{ij}G^\rho_{NN\to NN}$ $(ij = VV, VT, TV, TT)$ at $t=0$ for correlated $\pi\pi$ and $K\overline{K}$ exchange in the NN channel. (The meaning of the several rows is given in the text.)

${}^{ij}G^{\rho}_{NY\to NY'}/4\pi \quad (ij = VV, VT, TV, TT)$								
	$N\Sigma$				$N\Lambda \to N\Sigma$			
	VV	VT	TV	TT	VV	VT	TV	TT
full model	1.64	1.92	8.95	10.15	−0.15	3.97	−0.81	21.42
without baryon exchange	1.03	1.12	4.34	4.70	0.00	1.86	0.00	7.83
subtraction for OBE model	1.53	1.52	9.87	9.85	−0.52	4.54	−1.61	25.60

Table 7: The same as Tab. 6, for the hyperon-nucleon channels $N\Sigma$ and $N\Lambda$-$N\Sigma$.

${}^{ij}G^{\rho}_{AB\to AB}/4\pi \quad (ij = VV, VT, TV, TT)$							
	$\Sigma\Sigma$			$N\Xi$			
	VV	VT, TV	TT	VV	VT	TV	TT
full model	2.78	3.14	4.02	0.87	−1.92	4.71	−10.38
without baryon exchange	2.06	2.23	2.42	0.52	−1.05	2.17	−4.43
subtraction for OBE model	3.09	2.64	4.38	0.99	−2.30	5.40	−12.09

Table 8: The same as Tab. 6, for the baryon-baryon channels with strangeness $S = -2$.

$G^{\sigma}_{AB\to AB}/4\pi(\alpha_p)$							
	α_p	NN	$N\Lambda$	$N\Sigma$	$\Lambda\Lambda$	$\Sigma\Sigma$	$N\Xi$
	0.35	5.86	2.83	2.52	1.49	1.69	1.21
ps	0.40	5.87	2.82	2.58	1.52	1.72	1.19
	0.45	5.90	2.84	2.70	1.59	1.80	1.19
	0.35	6.00	3.36	3.16	2.04	2.37	1.64
pv	0.40	6.00	3.31	3.28	2.04	2.46	1.57
	0.45	6.00	3.31	3.45	2.10	2.62	1.54

Table 9: Effective σ coupling strengths $G^{\sigma}_{AB\to AB}(t = 0)$ depending on the $F/(F+D)$-ratio α_p. ps (pv) indicates that $SU(3)$-symmetry is assumed for the pseudoscalar (pseudovector) coupling constants $g_{BB'\mu}$ ($f_{BB'\mu}$). The column 'ps, $\alpha_p = 0.40$' agrees with the results of the full model in Tab. 5.

	NN	$N\Lambda$	$N\Sigma$	$\Lambda\Lambda$	$\Sigma\Sigma$	$N\Xi$
$\pi\pi + K\overline{K}$	1	0.49	0.41	0.26	0.30	0.19
OBEPT [10] & Jülich A [3]	1	0.45	0.63	0.34	0.66	
Nijmegen D [4]	1	1	1	1	1	1
Nijmegen F [5]	1	0.74	0.61	0.55	0.37	0.41
Nijmegen SC [6]	1	0.58	0.45	0.34	0.20	0.10

Table 10: Strength of the σ-like contributions (at $t = 0$) to the various baryon-baryon interactions relative to the NN channel. In case of the dispersiontheoretic result for $\pi\pi$ and $K\overline{K}$ exchange only those uncorrelated contributions are subtracted which are generated already in the s-channel by the iteration of an OBE potential (cf. Tab. 5). In case of the OBE models the numbers are extracted from the coupling constants of the σ- (OBEPT & Jülich A with inclusion of form factors and averaging over the several isospin channels) or the ϵ-meson (Nijmegen models and the $\Xi\Xi\epsilon$ coupling from $SU(3)$).

	NN		$N\Sigma$		$N\Lambda \to N\Sigma$	
	VV	TT	VV	TT	VV	TT
$\pi\pi + K\overline{K}$	0.28	28.87	1.25	8.07	−0.43	20.97
OBEPT [10] & Jülich A [3]	0.50	18.57	0.79	8.87	0	9.84
Nijmegen D [4]	0.35	23.20	0.71	15.51	0	17.82
Nijmegen F [5]	0.63	27.34	1.25	28.76	0	14.96
Nijmegen SC [6]	0.79	14.16	1.59	7.79	0	11.85

Table 11: Strength of the ρ-like contributions (at $t = 0$) to the interaction in the various baryon-baryon channels with $S = 0, -1$. The values for $\pi\pi$ and $K\overline{K}$ exchange correspond to the effective ρ coupling strengths in the third row of Tabs. 6–8 times the form factors for effective ρ exchange. For the OBE models the values are determined from the coupling constants of the ρ meson (in case of OBEPT and Jülich A under consideration of form factors).

Figure 1: Two-pion exchange in the nucleon-nucleon interaction: a) iterative boxes, b) crossed boxes, c) correlated two-pion exchange. The iterative box with an NN intermediate state is generated in the scattering equation by iterating the one-pion exchange. In OBE models all other contributions are parametrized by σ_{OBE} and ρ exchange. In the Bonn NN potential [10] the uncorrelated contributions a) and b) are evaluated explicitly whereas the correlated $\pi\pi$ exchange is parametrized by σ' and ρ exchange.

Figure 2: Two-pion and two-kaon exchange in the baryon-baryon process $A + B \to C + D$. The unshaded ellipse denotes the direct coupling of the two pseudoscalar mesons $\mu\overline{\mu} = \pi\pi, K\overline{K}, \overline{K}K$ to the baryons without any correlation effects (cf. Fig. 3). The shaded circle in the lower diagram for the correlated exchange stands for the full off-shell amplitude of the process $\mu\overline{\mu} \to \mu'\overline{\mu'}$.

Figure 3: Microscopic model for the $B\overline{B'} \to \pi\pi, K\overline{K}$ Born amplitudes. The solid lines denote (anti-)baryons, the dashed lines the pseudoscalar mesons $\pi\pi$ or $K\overline{K}$. The sum over exchanged baryons X contains all members of the $J^P = \frac{1}{2}^+$ octet and the $J^P = \frac{3}{2}^+$ decuplet which can be exchanged in accordance with the conservation of strangeness and isospin.

Figure 4: Two-particle scattering process.

Figure 5: Model for the $B\overline{B'} \to \mu\overline{\mu}$ amplitudes ($\mu\overline{\mu} = \pi\pi, K\overline{K}$).

Figure 6: Born amplitudes included in the model of Ref. [17] for the $\pi\pi - K\overline{K}$ interaction.

Figure 7: Contributions to the Born amplitudes $\Sigma\overline{\Sigma} \to \pi\pi, K\overline{K}$.

Figure 8: $\pi\pi$ phase shifts in the σ and ρ channel and the corresponding inelasticity in the σ channel. (From Ref. [17].)

Figure 9: $N\overline{N} \to \pi\pi, K\overline{K}$ helicity amplitudes in Frazer-Fulco normalization. The solid lines are the results of our microscopic model. The squares denote the quasiempirical data [18, 19] obtained by analytic continuation of the πN and $\pi\pi$ scattering amplitudes.

Figure 10: $N\overline{N} \to \pi\pi, K\overline{K}$ Born amplitudes.

Figure 11: $\Sigma\overline{\Sigma} \to \pi\pi, K\overline{K}$ Born amplitudes.

Figure 12: $\pi\pi$ and $K\overline{K}$ amplitudes in the σ channel ($J = I = 0$) as a function of the squared c.m. energy t. The dashed (solid) line represents the real (imaginary) part of the amplitude calculated with the fieldtheoretical model of Ref. [17].

Figure 13: Helicity amplitudes $T^J_\pm$ for $N\overline{N} \to \pi\pi$ (solid) and $N\overline{N} \to K\overline{K}$ (dotted). For $J = 1$ the small $N\overline{N} \to K\overline{K}$ amplitudes are suppressed. The dashed lines in the σ channel denote the $N\overline{N} \to \pi\pi$ amplitudes, that are calculated without the $N\overline{N} \to K\overline{K}$ Born amplitudes. In the ρ channel the dash-dotted lines are obtained if only the ρ-pole graphs is included in the $N\overline{N} \to \mu\overline{\mu}$ Born amplitudes.

Figure 14: $\Sigma\overline{\Sigma} \to \pi\pi, K\overline{K}$ helicity amplitudes $T^J_\pm$. For the meaning of the curves, see Fig. 13; in addition, the $\Sigma\overline{\Sigma} \to K\overline{K}$ amplitudes are included for $J = 1$.

Figure 15: $N\overline{N} \to \pi\pi$ amplitude T^0_+ calculated with the full model in the energy range around the $K\overline{K}$ threshold (dotted). The solid (dashed) line denotes the real (imaginary) part of T^0_+.

Figure 16: Spectral function $\rho^\sigma_S(t)$ for the scalar component of correlated $\pi\pi$ and $K\overline{K}$ exchange in the scalar-isoscalar channel of various baryon-baryon processes:

a) NN (solid), $N\Lambda$ (short dashed), $N\Sigma$ (dotted);
b) $\Lambda\Lambda$ (long dashed), $\Sigma\Sigma$ (dash-dotted), $N\Xi$ (dash-double-dotted).

Figure 17: Spectral function $\rho^\sigma_S(t)$ in the NN (solid) and $N\Sigma$ (dotted) channel derived with the full model (cf. Fig. 16). If the contributions of the $B\overline{B'} \to K\overline{K}$ Born amplitudes are neglected the dashed (NN) and the dash-dotted lines ($N\Sigma$) are obtained.

Figure 18: Spectral functions $\rho^\rho_i(t)$ ($i = S, V, T, 6$) for the contribution of correlated $\pi\pi$ and $K\overline{K}$ exchange in the ρ channel to the NN (solid), $N\Sigma$ (dotted), $\Sigma\Sigma$ (dash-dotted) and $N\Xi$ interaction (dash-double-dotted) as well as to the $N\Lambda - N\Sigma$ transition amplitude (short dashed).

Figure 19: NN spectral functions $\rho^\sigma_S(t)$ and $\rho^\rho_i(t)$ ($i = S, V, T$). The solid lines are derived with the microscopic model of Sect. 2.4 for the $B\overline{B'} \to \mu\overline{\mu}$ amplitudes. The results obtained with the quasiempirical $N\overline{N} \to \pi\pi$ amplitudes of Refs. [18, 19] are denoted by the dashed lines.

Figure 20: Effective σ coupling strengths $G^\sigma_{AB\to AB}/(4\pi)$ for correlated $\pi\pi$ and $K\overline{K}$ exchange as a function of the squared 4-momentum transfer $t < 0$ in the baryon-baryon channels: NN (solid), $N\Lambda$ (short dashed), $N\Sigma$ (dotted), $\Lambda\Lambda$ (long dashed), $\Sigma\Sigma$ (dash-dotted) and $N\Xi$ (dash-double-dotted).

Figure 21: Effective strength of the NN interaction due to correlated $\pi\pi$ and $K\overline{K}$ exchange in the σ and ρ channel as a function of the 4-momentum transfer $t < 0$. Shown are $g^2_{\sigma NN} \equiv G^{\sigma}_{NN\rightarrow NN}$, $g^2_{\rho NN} \equiv {}^{VV}G^{\rho}_{NN\rightarrow NN}$, $f^2_{\rho NN} \equiv {}^{TT}G^{\rho}_{NN\rightarrow NN}$ and $f_{\rho NN}/g_{\rho NN} \equiv [{}^{TT}G^{\rho}_{NN\rightarrow NN}/{}^{VV}G^{\rho}_{NN\rightarrow NN}]^{1/2}$ including the form factors. The solid (dotted) line is derived from the microscopic model for correlated $\pi\pi$ and $K\overline{K}$ exchange using $t'_{max} = 120m^2_\pi$ ($t'_{max} = 50m^2_\pi$, only for $g^2_{\sigma NN}$). The dashed line follows from the quasiempirical $N\overline{N} \rightarrow \pi\pi$ amplitudes [18, 19]. The effective strength of σ' and ρ exchange in the Bonn potential [10] is denoted by the dash-dotted line.

Figure 22: The σ-like part of the NN on-shell potential in various partial waves as a function of the kinetic energy in the laboratory system. The solid line is derived from our microscopic model for correlated $\pi\pi$ and $K\overline{K}$ exchange (with $t'_{max} = 120m^2_\pi$). The dotted lines are obtained if this dispersiontheoretic result is parametrized by σ exchange and the coupling strength $G^{\sigma}_{NN\rightarrow NN}(t)$ is subsequently set to the constant value at $t = 0$ The dispersiontheoretic calculation using the quasiempirical $N\overline{N} \rightarrow \pi\pi$ amplitudes of Refs. [18, 19] gives the dashed lines. Finally, the dash-dotted lines correspond to σ' exchange used in the Bonn potential [10].